Is Managed Futures an Asset Class?
The Search for the Beta of Commodity Futures

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Abstract

This paper investigates potential sources of return to speculators in the commodity futures market. Initially, we focus on the "classic arbitrage model" based on the theories of Keynes (1930), Kaldor (1939), Hicks (1939, 1946), Working (1948) and Brennan (1958). Next our study examines the "simplified arbitrage model" which references the term structure of the futures price curve and provides rationale for a structural risk premium known as the roll return. We then introduce our theory of "roll yield permutations" which is derived from integrating the futures price curve with the expected future spot price variable. Last, we investigate Spurgin's (2000) "hedging response model" from which asymmetric hedging response functions transfer risk premia to speculators.

Our research indicates that these models have inherent shortcomings in being able to pinpoint a definitive source of structural risk premium within the complexity of the commodity futures markets. We hypothesize that the classic arbitrage pricing theory contains circular logic, and as a consequence, its natural state is disequilibrium, not equilibrium. We extend this hypothesis to suggest that the term structure of the futures price curve, while indicative of a potential roll return benefit, in fact implies a complex series of roll yield permutations. Similarly, the hedging response function elicits a behavioral risk management mechanism, and therefore, corroborates social reflexivity. Such models are inter-related and each reflects certain qualities and dynamics within the overall futures market paradigm.

With respect to managed futures, it is an observable materialization of behavioral finance, where risk, return, leverage and skill operate un-tethered from the anchor of an accurate representation of *beta*. In other words, it defies rational expectations equilibrium, the efficient market hypothesis and allied models—the CAPM, arbitrage pricing theory or otherwise—to single-handedly isolate a persistent source of return without that source eventually slipping away.

Keywords: Futures Market, Commodities, Managed Futures, Backwardation, Contango, Risk Premia, Futures Price Curve, Roll Return, Convenience Yield, Capital Asset Pricing Model (CAPM), Alpha, Beta, Arbitrage Pricing Theory, Hedging Pressure Hypothesis, Rational Expectations, Equilibrium

JEL Classification: B23, C53, C68, D41, D58, D82, D84, E12, G11, G13, Q11, Q41

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I. Introduction: Alpha and Beta Quandary

It is difficult to be an investor nowadays and avoid being marketed 'alpha,' something which is intended to measure a manager's skill-based returns. The term, which is derived from statistics, is the byproduct of a linear regression that relates an observed variable y to some factor x, resulting in the equation $\alpha = y - \beta x - \varepsilon$, where α (alpha) represents the intercept, β (beta) represents the slope, and ε (epsilon) represents a random error term. In finance, alpha is defined as the excess return that results from active portfolio management adjusted for the risk of a comparable risky asset or opportunity set. In industry practice, the term has become a marketing devise.

As Schneeweis (1999) pointed out in his article "Alpha, Alpha, Whose got the Alpha?" it is inappropriate to compare investment returns to the S&P 500, or any other benchmark, unless the investment strategy being analyzed responds to the same return drivers of the S&P 500 or the cited benchmark. Similarly, it is inappropriate for a manager to make a claim of positive alpha simply because investment returns are greater than the risk free rate, unless the portfolio is risk-free. Accordingly, investors should first be concerned with the appropriateness of the reference beta.

Conventional investment theory states that when an investor constructs a well-diversified portfolio, the unsystematic sources of risk are diversified away leaving the systematic or non-diversifiable source of risk as the relevant risks. The capital asset pricing model (CAPM), developed by Sharpe (1964), Lintner (1965) and Black (1972) [zero-beta version], asserts that the correct measure of this riskiness is its measure known as 'beta,' and that the risk premium per unit of riskiness is the same across all assets. Effectively, this coefficient is an index of an asset's correlated volatility relative to the volatility of the overall market. Consequently, given the risk-free rate and the beta of an asset, the CAPM should be able to predict the expected risk premium for that asset, and correspondingly the expected return as well.

The above explanation is textbook. However, unbeknownst to most investors, there has been a long running deliberation in academic circles on the CAPM and other pricing models, even within the milieu of traditional investments. The evolution of this dispute is thoroughly documented by Jagannathan and McGrattan (1995) in their article, "The CAPM Debate" published by the Federal Reserve Bank of Minneapolis Quarterly Review. The following inquiry, paraphrased directly from Jagannathan and McGrattan's article, describes the crux of the issue with respect to how accurately the CAPM performs in determining beta when empirically tested:

> The CAPM was developed, at least in part, to explain the difference in risk premium across assets. According to the CAPM, these differences are due to differences in the riskiness of the return on the assets. When the CAPM assumptions are satisfied, everyone in the economy will hold all risky assets in the same proportion. Hence, the betas computed with reference to every individual's portfolio will be the same, and one might as well compute betas using the market portfolio of all assets in the economy. The CAPM predicts that the ratio of the risk premium to the beta of every asset is the same. That is, every investment opportunity provides the same amount of compensation for any given level of risk, when beta is used as the measure of risk. [Accordingly, if] expected returns vary across assets, [it is] only because the assets' betas are different. [Therefore,] one way to investigate whether the CAPM adequately captures all important aspects of reality is to test whether other asset-specific characteristics can explain the cross-sectional differences in average returns that are unrelated to cross-sectional differences in beta. [As a result,] in empirical evaluations of the CAPM, researchers want to know… if beta is the only characteristic that matters.[1]

The earliest empirical studies of the CAPM, including that of Black, Jensen and Scholes (1972) and Fama and MacBeth (1973) concluded that the data was consistent with the predictions of the CAPM. Banz (1981), however, challenged the CAPM and found that a significant factor, firm size, explained cross-sectional variation in average returns on a collection of assets better than beta. The reaction to Banz's findings was that, while the data showed some systematic deviations, such anomalies were not economically important enough to reject the CAPM outright. This view was challenged by Fama and French (1992), who concluded that "Banz's findings may be economically so important that it questions the validity of the CAPM," and that explanatory variables such as the book-to-market equity ratio better explained cross-sectional variation in average asset returns. In response, a series of counter-challenges to Fama and French, while supportive of the CAPM, revealed a variety of other potential statistical concerns including noisy data, sample period effect and survivorship bias.

Nonetheless, Fama and French's (1992) core challenge has remained that the "resuscitation of the [Sharpe 1964, Lintner 1965, Black 1972] model requires that a better proxy for the market portfolio… leaves β (beta) as the only variable relevant for explaining average returns." Given that approximately one-third of non-governmental tangible assets in the United States are owned by the

[1] Paraphrased from Jagannathan, Ravi; McGrattan, Ellen R. 1995. "The CAPM Debate" *Federal Reserve Bank of Minneapolis Quarterly Review* Vol. 19, No. 4, Fall 1995, pp. 2-17.

corporate sector, and only one-third of these corporate assets are financed by equity, it seems intuitive to abandon the assumption that the broad stock market indexes are an adequate proxy for beta. As a consequence, academic and institutional focus has shifted to alternative asset pricing methods including the development of 'multi-factor models.' These additional factors have extended beyond the use of broad stock market indices as "a reasonable proxy for the return on the true market portfolio of all assets in the economy," in order to capture intangible assets such as human capital and variables such as business cycles, as well as other attributes.

Meanwhile, in a direct response to Fama and French's (1992) challenge, Jagannathan and Wang (1993) theorized that "…the lack of empirical support for the CAPM may be due to the inappropriateness of some assumptions made to facilitate the empirical analysis of the model. Such an analysis must include a measure of the return on the aggregate wealth portfolio of all agents in the economy." By following Jagannathan and Wang's thought process to its logical conclusion, taking into consideration the globalization and integration of the world's economies, we have extended this definition of 'true market portfolio' or 'true beta' to encompass the 'aggregate wealth portfolio of all the agents in the *global* economy,' something related to the "gross global product" (GGP).[2] The advantage of this archetype is that it is a closed box system, yet one which encompasses all possible economic activities and variables that exists in the *real world*.

Financial institutions have not been left behind by these evolving academic theories. Index creation and benchmarking has become standard fare, and since the introduction of 'exchange traded funds' (ETFs), a veritable industry has developed around the 'multiple beta' concept. But by no means has the plethora of these instruments captured every aspect of the *aggregate wealth portfolio of all agents in the global economy*, although at the current pace of ETF development it would seem that this is the undeclared objective. If one assumes that true beta is equivalent or linked to GGP, it becomes evident that the conventional benchmarks for beta, such as the S&P 500 Index, Dow Jones Wilshire 5000 Index, Lehman Brothers Aggregate Bond Index, MSCI Europe Australasia and Far East Index, S&P GSCI™ Commodity Index, etc., and even economic indicators such as the Consumer Price Index, Employment Report, Industrial Production, etc. all *represent arbitrary slices or aspects of true beta*. Further, the overlapping and inherently reflexive nature of these benchmarks' intrinsic attributes becomes apparent.

This legacy backdrop is the principal context which gives impetus to the notion of 'exotic betas.' The term, a recent addition to the investment lexicon which evolved from ideas advanced by

[2] The World Bank. Global Citizen's Handbook: Facing Our World's Crises and Challenges. Collins. 2007

proponents of alternative investments, suggests that certain alternative investment assets and/or strategies, representing commonly pursued market paradigms, can be identified, tracked and replicated employing a predefined "passive" approach/model similar to traditional index construction. In fact, it is the very existence of the idea of exotic betas which is fueling the demand for tailored commodity indices,[3] such as Goldman Sachs "smart indexes" like *GS Connect S&P GSCI Enhanced Commodity Total Return Strategy Index Exchange Traded Note* (Symbol: GSC), which uses seasonal and other pricing trends. This leaves open the question as to whether institutions, through sophisticated financial engineering, can truly capture in a passive way all possible sources of return in the global economy. *Or*, does some aspect which the industry loosely calls alpha (i.e., skill-based returns) always remain outside the grasp of such institutions' "arbitrary" models of beta? In either case, it is from the presupposition—that the 'true market portfolio' is defined as *the aggregate wealth portfolio of all the agents in the global economy*—where we begin our investigation into the problem of how to determine the most appropriate beta proxy for commodity futures.

II. Framing the Futures Market Beta Debate

"By 'uncertain' knowledge, let me explain, I do not mean merely to distinguish what is known for certain from what is only probable… The sense in which I am using the term is that in which the prospects of a European war is uncertain, or the price of copper and the rate of interest twenty years hence… About these matters there is no scientific basis on which to form any calculable probability whatever. We simply do not know." – John Maynard Keynes (1937)

It is assumed that organized futures markets provide important economic benefits. This premise, that properly functioning futures markets serve a valuable economic purpose, is validated by government policy.[4] The secondary benefit provided by the futures market is that it functions as a mechanism for transparent price discovery and liquidity, which therefore mitigates price volatility. The primary benefit provided by these markets, however, is that it allows commercial producers, distributors and consumers of an underlying cash commodity to hedge.[5] This reduces the risk of adverse price fluctuations that may impact business operations, which in turn theoretically results in increased 'capacity utilization.'[6] Hence, it follows that the reallocation of risk affords a reduction in

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[3] As of 2006, US$95 billion was estimated to be invested in long-only commodity indexes [Piotrowski (2006)] versus US$6 billion invested in such indexes in 1996 [Meir and Demmler (2006)]

[4] In testimony on November 2, 2005 before the Committee on Energy and Commerce United States House of Representatives, Reuben Jeffery III, Chairman U.S. Commodity Futures Trading Commission stated that "Futures markets play a critically important role in the U.S. economy."

[5] By using futures or forward contracts to hedge, a producer, distributor or consumer of an underlying asset can establish a temporary substitute for a cash market transaction that will be made at a future date.

[6] Capacity utilization is a metric used to measure the rate at which potential output levels are being met or used. Capacity utilization rates can also be used to determine the level at which unit costs will rise.

prices of the underlying commodity because businesses need not offset adverse price change risk with increased margins on products or services.

These economic benefits should be realized by the businesses that utilize futures markets for bona fide hedging purposes. For that reason, we assume that such factors (i.e., capacity utilization, price discovery, price liquidity and reduced price volatility, etc.) are reflected in the economy and therefore in business earnings. Since businesses fall into the category of traditional investments, and the beta proxies for stocks and bonds are well represented, this segment of 'true beta' is not the focus of our research. Rather, our investigation starts with established precepts that form the basis of academic studies which attempt to model the sources of return in the futures market. That is, the "beta of commodity futures" emanates from capturing the 'risk premia' hedgers supposedly offer speculators for assuming the risk that these aforementioned businesses are trying to offset.

Till (2007) notes that there are seven such themes influencing commodity pricing theory: (1) the insurance role of commodity futures contracts which emphasizes the role of the speculator; (2) the theory of storage, which emphasizes the behavior of the inventory holder and commercial hedger; (3) the net-hedging-pressure hypothesis, which encompasses the behavior of both classes of participant; (4) the statistical behavior of commodity futures prices; (5) the attempt to reconcile commodity futures returns with the CAPM; (6) the role of commodities in a strategic asset allocation; and (7) the importance of yields as a long-term driver of commodity returns.[7]

The insurance-like context was first proposed by Keynes (1923, 1930) in his theory of 'normal backwardation.' Essentially, Keynes believed that hedgers have to pay speculators a risk premium to convince them to accept their risk. A key attribute of this theory is the concept of 'congenital weakness' on the demand side for commodities. As expounded by Hicks (1939, 1946), consumers are generally better positioned to choose amongst delivery alternatives as well as time their purchases; whereas producers have operational constraints, are more exposed to commodity price fluctuations, and for that reason are under more pressure to hedge. Hicks' ideas directly relate to another aspect of Keynes' commodity hypothesis involving a producer's current commitment to deliver a commodity in the future superseding the increased reward that could result from selling that commodity in the future. Accordingly, the current expectation of the future spot price (which is in actuality an unknown) is theoretically driven down because the commodity is held back from the market and kept in storage. As described by Kaldor (1939),

[7] Sourced from Till, Hilary. 2007. "Part I of A Long-Term Perspective on Commodity Futures Returns: Review of the Historical Literature," Intelligent Commodity Investing, (Till, and Eagleeye, Ed.), Published by Risk Books, a Division of Incisive Financial Publishing, Ltd., pp. 39-82.

holding back a commodity in storage is referred to as a 'convenience yield,' and together with congenital weakness forms the basis of the phenomenon known as 'backwardation.' These concepts are now part of mainstream thinking. Nevertheless, the legacy of empirical tests using a variety of asset pricing models, including CAPM/C-CAPM, hedging-pressure hypothesis, or arbitrage pricing theory, have produced contradictory results as to whether there is, in fact, positive expected returns from speculating in the commodity futures market, or whether investment in such markets provide only zero systematic risk—that is, *a zero sum game*.

Dusak (1973) was the first to investigate the beta of the futures market using the CAPM and found zero systematic risk. However, Bodie and Rosansky (1980), Fama and French (1987) and Hirshleifer (1988), who combined both the CAPM and the hedging-pressure hypothesis, all found positive expected returns which supported the theory of normal backwardation. Ehrhardt, Jordan and Walking (1987), on the other hand, found no risk premium using the arbitrage pricing theory, while Miffre (2000, 2003), using a combination of asset pricing models, more recently concluded in favor of normal backwardation. Yet only a decade earlier, Kolb (1992) found that only seven out of twenty-nine commodity and financial futures support normal backwardation. As documented by Allen, Cruickshank, Morkel-Kingsbury and Souness (1999), "there is no consistent evidence about the existence of normal backwardation despite a long tradition of research which dates back to Keynes (1930), Hardy (1940), Working (1948, 1949), Houthakker (1957), Telser (1958, 1967), Cootner (1960, 1967), Rockwell (1967) and Dusak (1973)."As noted by Greer (1997), the inherent problem with reconciling the CAPM to investment in commodities may be that these 'real assets' are not capital assets but instead 'consumable/transformable assets' with unique attributes. At the very least, the catalogue of academic research in this area is extensive and continues to grow.[8]

One reason propelling academic interest to search for sources of return in commodity futures is the conundrum that trading futures is a zero-sum game resulting in symmetric open interest. Paraphrasing Spurgin (2000), the argument against speculating in futures is based on the premise that—if there were excess returns to speculative capital in futures trading, assuming there are participants such as risk averse hedgers willing to lose money over time, then since barriers to

[8] The following list catalogues post-Dusak (1973) studies identified in this area of research: Breeden (1980); Bodie and Rosansky (1980); Rolfo (1980); Newbery and Stiglitz (1981); Anderson and Danthine (1983); Carter, Rausser and Schmitz (1983); Baxter, Conine and Tamarkin (1984); Britto (1984); Marcus (1984); Raynauld and Tessier (1984); Jagannathan (1985); Chang (1985); Park (1985); Fama and French (1987); Erhardt, Jordan and Walking (1987); Hartzmark (1987); Hirshleifer (1988); Young and Boyle (1989); Bessembinder (1992); Bessembinder and Chan (1992); Kolb (1992); Kolb (1996); de Roon, Nijman and Veld (1998); Allen, Cruickshank, Morkel-Kingsbury and Souness (1999); de Roon, Nijman and Veld (2000); Francis (2000); Miffre (2000); Lee (2003); Ebrahim and Rahman (2004); Dietz, Good, Irwin and Shi (2005); de Roon and Szymanowska (2006); Erb and Harvey (2006); Szymanowska, Goorbergh, Nijman and de Roon (2006); Gorton and Rouwenhorst (2006).

entry for trading futures is low, so much capital would flow to this industry that returns would be driven to zero over time, and as a result returns would be spread so thinly that economic profits would not be possible. Interestingly, contemporaneous and parallel with several years of increased investment in commodities, and corresponding institutionalization of the space, there seems to be a recent rash of industry papers supportive, if not presumptive, of the idea of a 'structural risk premium' in the commodity futures markets. Forgotten is the counter-argument that trading commodity futures is a zero-sum game with a long-term symmetric outcome, while leaving the potential for short-term asymmetric winners and losers.

Nevertheless, empirical research is still unable to bridge the disparity. A recent study by Erb and Harvey (2006) posits the following question, "For investors considering a long-only investment in commodity futures: how can a commodity futures portfolio have 'equity-like' returns when the average returns of the portfolio's constituents have been close to zero?" As noted by Ebrahim and Rahman (2004), who "echo" Bray (1992), Sheffrin (1996) as well as Malliaris and Stein (1999), "this discrepancy between theoretical assertions and empirical behavior is a puzzle. Is there something missing in the theory?" We think the answer is *yes*! In line with Jagannathan and Wang's (1993) premise that the lack of empirical support for the CAPM may be due to inappropriate assumptions, we propose that imperfect assumptions underlying recent empirical studies which analyze for sources of return in the commodity futures market, specifically those which advocate the idea of a 'roll return' or 'roll yield,' has resulted in perpetuating deficient theories into the investor mindset. To support our view, the following section of our working paper, "Models of Equilibrium or Disequilibrium," probes the flawed thought process inherent with "current market convention" which defines 'backwardation' and 'contango' as related to the "term structure of the futures price curve," as opposed to the ideas first proposed by Keynes et al. which relates normal backwardation to the 'expected futures spot price,' which is an unknown.

Further, from the perspective of most *real-life* speculators, academic theoretical models have little to do with how most practitioners (i.e., traders) actually speculate in the futures markets. Additionally, we note it is dangerous to extrapolate past performance into multi-factor regression analysis as a means to predict future outcomes. It is a well understood tenet of professional systematic traders that expanding the number of factors, conditions and variables increases the likelihood of curve-fitting to historical data, also referred to as 'over-optimization.' Moreover, we suggest that commodity asset pricing models, which are conventionally regarded as validation for persistent and replicable sources of return in the commodity futures markets, may be widely misunderstood. There is no doubt that the futures markets offer vicarious economic benefits, such as price discovery, price liquidity, reduced price volatility and therefore increased capacity utilization. But again, such attributes benefit the businesses that utilize the futures markets, as well as the economy as a whole, not necessarily those economic agents called speculators. We therefore contend that index vehicles based on commodity assets will prove over the long run (beyond the current secular bull market in commodities) to *not* be the reliable and consistent source of positive expected returns as is proposed by others.

Admittedly, models are only an abstraction from reality. Expecting such models to be exactly right is unreasonable, and it is generally understood that neoclassical economic models have inherent limitations related to the analysis of markets within the context of rational equilibrium systems. Such systems are based on perfect competition, assume that the economy is stable, and that markets naturally return to equilibrium after a disturbance. Hence, such models maximize utility and/or profits in a world of constraints based on the choices of "rational" economic agents. By definition then, these models relegate speculators to the role of that very agent which maintains equilibrium. Yet a survey of *real-life* speculators reveals that these practitioners do not as a general rule use academic models in their day-to-day speculative trading decisions.[9] Paradoxically, this same group plays a key influence upon the selfsame futures data from which these models are constructed. So if the data series is assumed to represent equilibrium and "the future is merely the statistical reflection of the past,"[10] then one could inversely argue that perfect competition and rational expectations minimizes these models' usefulness as a mechanism from which to make speculative decisions. In other words, rational expectations compel such models to simply validate that current price data is equal to equilibrium; unless the opposite is true—that markets are in fact imperfect and rational expectations is untenable, which in turn undermines the veracity of these models.

However, we are not saying that commodity futures pricing models are completely erroneous. Rather, while conceived and constructed using 'rational expectations equilibrium' and so interpreted within that framework, the models investigated arguably imply 'disequilibrium' and 'social reflexivity.' Additionally, these models do not operate to the exclusion of the other, nor exclusively from each other; instead, such models are inter-related and each reflect certain aspects and dynamics within the overall futures market paradigm. What these models convey is an insightful understanding, provided one accepts that in the real world agents are irrational, that markets drift from disequilibrium to equilibrium and back, and inputs/outputs are reflexive. Hence, we posit that the combination of commodity futures pricing models in fact support a *post-Keynesian* view that the world is messy and uncertain.

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[9] An exception to this assertion is the Black-Scholes option pricing model, which is widely used by practitioners.

[10] Davidson, Paul (1982). "Is Probability Theory Relevant for Uncertainty? A Post Keynesian Perspective" *The Journal of Economic Perspectives*, Vol. 5, No. 1 (Winter, 1991), pp. 129-143

III. Models of Equilibrium or Disequilibrium?

Research Introduction and Premise

Our research investigates several models which explore the potential sources of return to speculators in commodity futures, including one of our own design which exemplifies the complexity of these markets. Initially we focus on the "classic arbitrage model," based on the theories of Keynes (1930), Kaldor (1939), Hicks (1939, 1946), Working (1948) and Brennan (1958), and which ensures convergence of the futures contract price with the current spot price, and from which the original definition of *backwardation* and *contango* was derived. Next we examine backwardation and contango in line with current market convention, which references the relationship between the nearby futures contract and such futures' subsequent contract months. This characteristic, which Till (2007) refers to as the 'term structure' of the 'futures price curve,' provides the underlying rationale for a 'structural risk premium' known as the 'roll return' or 'roll yield.'[11] We refer to this term structure as the "simplified arbitrage model," and look to further academic discussion by presenting our theory of "roll yield permutations." Last, our study looks at the "hedging response model" based on Spurgin's (2000) article "Some Thoughts on the Source of Return to Managed Futures," and from which he theorizes symmetric and asymmetric 'hedging response functions.' It is noted that Spurgin's model is rooted in Cootner's (1960, 1967) hedging-pressure hypothesis.

To minimize confusion we narrowed our analysis to futures based on commodity interests, and excluded futures based on financials assets (e.g., contracts based on equities such as the S&P 500, or fixed income such as the Eurodollar) which embody intrinsically different economic dynamics and return characteristics. It is also noted that evolving semantics in this area of financial research has created confusion, and that the definition of *backwardation* and *contango* as currently adopted by many academics and practitioners is often in line with Till's (2007) term structure of the futures price curve. Alternatively, we have taken the effort in this paper to distinguish these terms within the context of the classic arbitrage model versus the simplified arbitrage model in order to preserve conceptual subtleties, which in turn allowed us to develop and present our theory on roll yield permutations. In addition, we note that while the classic arbitrage model is directly derived from Keynes' commodity hypothesis, where (a) the spot price of a commodity normally exceeds its future price, and (b) the expected future spot price exceeds the current futures price, Keynes' "theory of normal backwardation," as clarified below, is considered for the purpose of our investigation to be a variation of the classic arbitrage model encompassing the "natural" constraints of congenital weakness and convenience yield.

[11] Empirical studies are not conclusive as to the persistence of a 'roll yield,' and spirited academic debate on this subject continues. In the 2006 *Financial Analysts Journal*, Gorton and Rouwenhorst (2006) argued that a structural risk premium is available across commodity futures contracts; while Erb and Harvey (2006) made a case for a strategy return based on how one weights and rebalances a commodity portfolio.

Classic Arbitrage Model

The "classic arbitrage model" focuses on the normal relation between the present state and future expectations of three variables: (i) the current spot price of an asset; (ii) the current futures or forward contract price of the underlying asset; and (iii) the expected spot price on delivery of the underlying asset sometime in the future. Let S_0 be the current spot price of the asset; F_t be the current price for future delivery of the underlying asset, and $E(S_t)$ be the expected spot price of the underlying asset on the delivery date. It is also noted that S_0 is a known variable equal to a price currently obtainable in the spot market for the underlying asset; F_t is a known variable equal to the current futures or forward contract price quoted on a futures exchange or over-the-counter market; but that $E(S_t)$ is an unknown variable which converts into S_0 at some future point in time.

There are two underlying speculative strategies (A) and (B), which in combination act to form a third strategy (C) that arbitrages for 'carrying charge' parity. These strategies are reviewed below, but before continuing, a description of the "theory of storage" is helpful. Carrying charge or the 'cost-of-carry' is based on the theory of storage first developed by Kaldor (1939) and Working (1948). The theory of storage assumes that holders of commodities incur a 'storage cost' for financing and storing inventories (including insurance and transportation), as well as a 'convenience yield' benefit of being able to use inventories the moment they are commercially needed. Storage cost, if dominant, is stated to be responsible for producing contango; while convenience yield, if dominant, is said to be the reason for backwardation. In combination, storage cost and convenience yield is expressed as the cost-of-carry which is derived from Kaldor's equation $F_t - S_0$ = storage costs + interest costs – convenience yield. Conversely, convenience yield = $S_0 - F_t$ + storage costs + interest costs. As a result, $E(S_t)$ should theoretically equal S_0 plus the cost-of-carry.[12]

(A) The first speculative strategy capitalizes on the implied relation between S_0 and $E(S_t)$. This relation is interposed by financing physical delivery of the commodity today, and selling the same commodity in the future at the then prevailing spot price. In this scenario, an equilibrium state is achieved when S_0 plus cost-of-carry is equal to $E(S_t)$. Hence, if S_0 plus cost-of-carry is theoretically $< E(S_t)$, then speculators could make a profit by taking physical delivery of a greater quantity of the commodity today (driving up the current spot price), with the intention of selling a greater quantity of the commodity in the future (driving down the expected spot price).[13] Assuming

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[12] Working, who examined futures spreads versus prevailing inventories, found that "carrying charges behave like prices of storage as regards their relation [sic] to the quantity of stocks held in storage." Accordingly, he defined the cost-of-carry as the "difference at a given time between prices of a commodity for two different dates of delivery," which relates to the futures price curve characteristic discussed later in this paper.

[13] The reason why the opposite trade is less likely to occur, besides it being difficult to borrow and short S_0, relates to Keynes theory of 'normal backwardation' which is explained in detail below.

perfect markets and rational expectations (i.e, market participants are risk neutral, know perfectly the cost-of-carry, and transaction costs are zero), this is supposedly the raison d'etre that enforces equilibrium between S_0 and $E(S_t)$. However, since $E(S_t)$ is in the future and therefore an unknown, this strategy is technically speculation, and for that reason determining the theoretical price of $E(S_t)$, if feasibly possible, ought to necessitate interpolation using at least one or more other references.

(B) The second speculative strategy capitalizes on the implied relation between F_t and $E(S_t)$. This relation is interposed by buying the futures contract today, taking physical delivery of the commodity when the contract expires, and at that time simultaneously selling the same commodity in the spot market. In this scenario, an equilibrium state is achieved when F_t net cost-of-carry is equal to $E(S_t)$. Hence, if F_t net cost-of-carry is theoretically $< E(S_t)$, then speculators could make a profit by purchasing the futures contract (driving up the futures contract price), with the intention of taking delivery and selling the commodity at the prevailing spot market price in the future (driving down the expected spot price). Additionally, since it is easy to short futures contracts, if F_t net cost-of-carry is theoretically $> E(S_t)$, then speculators could also make a profit by shorting the future contract (driving down the futures contract price), with the intention of making delivery by purchasing the commodity at the prevailing spot market price in the future (driving up the expected spot price). Assuming perfect markets and rational expectations, this is the raison d'etre that supposedly imposes equilibrium between F_t and $E(S_t)$. In fact, the function of F_t is to serve as *the mechanism* of 'price discovery' for $E(S_t)$. Therefore, if one accepts situations where F_t net cost-of-carry is either < or > $E(S_t)$, then by inference either: (i) the cost-of-carry assumption is incorrect and needs to be adjusted so that $F_t = E(S_t)$; (ii) the markets are imperfect and rational expectations is untenable which allows arbitrage opportunities to exist; or (iii) there is some other unknown factor or variable in this model which is causing an unexplained anomaly or error.

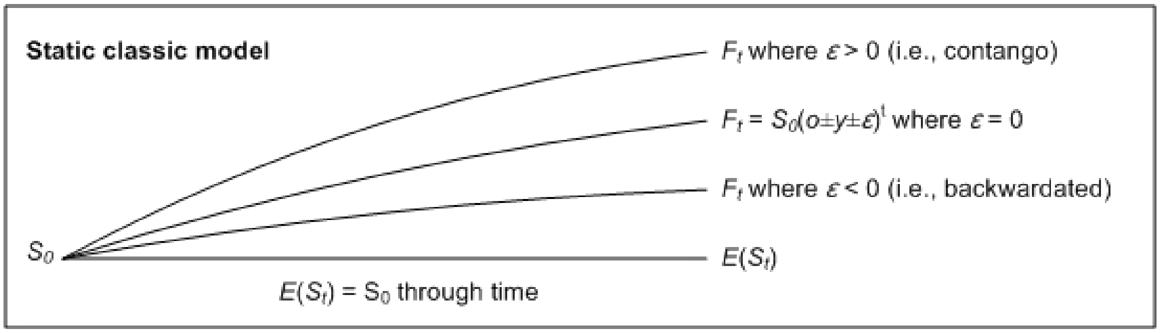
(C) As a result of speculative strategies (A) and (B), assuming perfect markets, rational expectations and arbitrage convergence, then F_t should equal $S_0(o + r - y)^t$, where o is the outlay on physical storage including facilities, insurance and interest; r is a marginal risk-aversion factor (applied to commodities without futures contracts); y is the 'convenience yield' which is expected to increase as inventories decrease; and t is the time to delivery of the underlying asset. This describes the third strategy (C) which arbitrages for 'carrying charge' parity; with the variables given based on Brennan's (1958) ideas, who reasoned that outlays are generally constant until facilities reach capacity, and then costs would increase as storage becomes limited. Brennan, who applied his research to actual market data, referred to $E(S_t) - S_0$ as the price spread, and inferred that $r - y$ was a result of $E(S_t) - S_0 - o$, where $E(S_t)$ is equal to F_t in lieu of commodities without futures contracts. Evidence of convenience yields at low inventory levels was also confirmed by Telser (1958) in his empirical study on futures price spreads and inventories from 1926 to 1954.

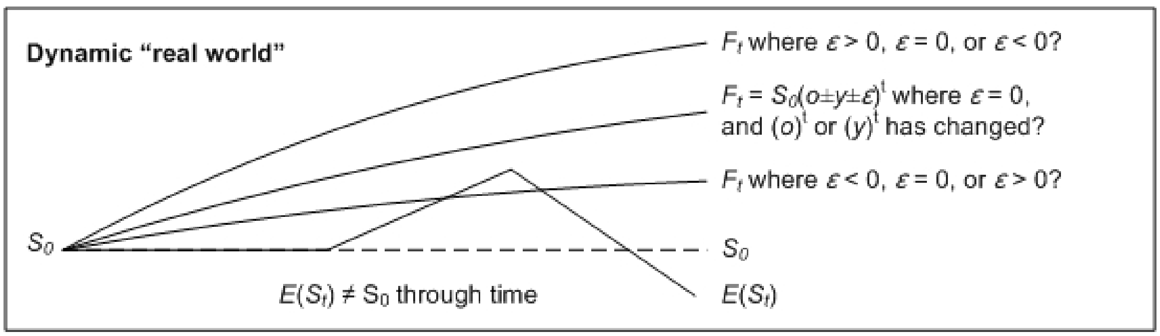
For reasons which shall become apparent, and also since o is readily determinable (i.e., derived from observable fundamentals) whereas r and y are inferred, we propose three alternative equations for solving for F_t based on S_0. First, the formula $F_t = S_0(o + r - y)^t$ can be simplified as $F_t = S_0(o - y \pm \varepsilon)^t$, where $\pm\, \varepsilon$ is a random error term over/under y, and where y is assumed to be determinable in keeping with tradition. However, it should be noted that Kaldor (1939) qualified his convenience yield benefit and storage theory equation as not applicable when hedgers are forward buyers (except for precious metals, this condition has not been thoroughly researched). We have taken the liberty of dubbing this condition "inconvenience yield" to counterpoint the well-established convenience yield condition.[14] Therefore, since convenience yield ($-y$) or inconvenience yield ($+y$) is inferred by netting o from $E(S_t) - S_0$, we can alternatively rewrite the carrying charge parity as $F_t = S_0(o \pm y \pm \varepsilon)^t$, where $\pm\, \varepsilon$ is a random error term over/under y, and where y is assumed to be determinable. This second version still presumes that $\pm y$ is a known value distinguishable from other pricing anomalies subject to arbitrage. Accordingly, our third variant of the cost-of-carry formula rewrites ε as a factor, such that $F_t = S_0(o - y \cdot \varepsilon)^t$, where ε is a random error *factor* from which $\pm y$ can be inferred, but is only determinable as a function of whether ε is either ≥ 1, or ≤ 1, or whether ε equals 0, in which case the cost-of-carry consists of storage outlay only without any convenience yield attribute. Why is the second and third variation of the carry charge parity equation more applicable than versions traditionally used?

Interestingly, the classic arbitrage model states that if one supposes arbitrage convergence is imperfect, and there exists the presence of economic stimulus which leads to either (a) backwardation or (b) contango market conditions, the model reasons it possible to have $E(S_t)$, which *in the real world* is an unknown, valued above or below $S_0(o + r - y)^t$. This is a point of paramount significance! The classic arbitrage model (C) allows for and generates circular math which is not *classically* explained. We try to resolve this issue by including in the $S_0(o \pm y \pm \varepsilon)^t$ and the $S_0(o - y \cdot \varepsilon)^t$ expressions, a random error term [or factor] which can either be: (a) negative [or a factor >1], if convenience yield dominates (backwardation), in which case F_t is $< E(S_t)$; or (b) positive [or a factor <1], if storage costs dominates (contango), in which case F_t is $> E(S_t)$. However, for either of these scenarios, the *causal relativity* (i.e., the outcome of a cause must always be determined/evaluated relative to a control condition) of S_0 and F_t to $E(S_t)$, while reflexive and part of the price discovery process, is not normally deliberated because of rational expectations. Yet, if one assumes imperfect markets and the potential for variability in the value of F_t, $E(S_t)$ or S_0, the consequential effect is circular logic. Further, if we accept that the market can be backwardated or contango, then the

[14] Given the emergence of China and other emerging economies, as well as the increasing acceptance of long-only commodities as an asset class, we anticipate that the 'inconvenience yield' concept will likely be studied more thoroughly in the coming years as this market characteristic potentially becomes persistent.

classic arbitrage model necessitates that either F_t or $E(S_t)$ is mispriced, or alternatively it is possible that S_0 may be mispriced. At the same time, it is not possible to have outcomes where only the carrying charge is indefinite based on a random error term or factor and not subsequently influence the valuation of either F_t, $E(S_t)$, S_0, or all three of these variables simultaneously. The graphs below help illustrate this conundrum using our $S_0(o \pm y \pm \varepsilon)^t$ version of the carrying charge equation.





Therefore, depending on whether one uses our second [or third] version of the cost-of-carry formula, the problem facing arbitrageurs with respect to scenario (C) is how to determine whether ε is zero [or ε = factor of 1] and either $(o)^t$ or $(y)^t$ assumptions have increased or decreased due to a change in fundamentals, or whether ε is greater or less than zero [or ε > or < factor of 1], and an arbitrage opportunity in fact exists. Since the model's logic is circular, the reflexive relationship between S_0 and $E(S_t)$ versus $(o)^t$, $(y)^t$ or ε, and between F_t and $E(S_t)$ versus $(o)^t$, $(y)^t$ or ε, as well as between F_t and S_0 versus $(o)^t$, $(y)^t$ or ε, should be iterated continuously in order to determine if either F_t or $E(S_t)$ is accurately valued or mispriced. Likewise, the question facing speculators with respect to either scenario (A) or (B) is deciding whether the market's current inferred cost-of-carry is correct, and from that one can correctly interpolate $E(S_t)$; or whether the market's inferred cost-of-carry is incorrect and a speculative opportunity exists. If yes, then the speculator can enter into a long or short S_0 cash commodity or F_t futures contract position, and take delivery or deliver the commodity at some point in the future in the case of S_0; or in the case of F_t, take delivery or deliver the commodity when F_t expires and $E(S_t)$ is converted into S_0.

Hence, for practitioners, applicability of the classic arbitrage model is largely dependent on whether rational expectations equilibrium is presumed to accurately represent how the *real world* works, or whether one can properly assume that markets are imperfect and rational expectations is untenable. If one sides with the latter argument, that markets are imperfect and rational expectations is untenable, then the classic arbitrage model, while rendered less useful for instigating trade decisions (or for that matter less capable as a methodology for empirical testing of backwardation or contango conditions), theoretically engenders the existence of price reflexivity. As a result, the slightest adjustment to any variable or expectation in the model could potentially cause S_0, F_t and/or $E(S_t)$ to skew in one direction or the other (rather than converge toward equilibrium) eventually resulting in an exponential change in prices. If, however, one believes in the assumption of perfect markets and rational expectations, then the definition of backwardation or contango within the context of the classic arbitrage model either: (i) is discordant because $E(S_t)$ is by both the model's definition and in the *real world* an unknown; or (ii) engenders reflexivity but where arbitrage convergence is assumed to be in perfect equilibrium and prices always reflect that "fact." This is where economic modeling gets philosophical.

To the first point (i), information about the future is costly; therefore an 'optimal forecast' may be the best not because it is accurate, but because it is too expensive to attain true accuracy. Further, Keynes himself noted that the fundamental uncertainty about the future is such that *no* expectations can be truly rational, although proponents of ergodic systems dismiss Keynes in this respect.[15] As to the second point (ii), it is noted that the efficient market hypothesis and rational expectations assume a predetermined equilibrium around which expectations are formed.[16] *In fact, these expectations determine the nature of the equilibrium attained, reversing the line of causation posited by rational expectation theorists*. A further problem, as premised by the Sonnenschein-Mantel-Debreu theorem, relates the application of rational expectations to aggregate behavior, given that assumptions about individual behavior do not carry over to aggregate behavior. According to Janssen (1993), even if all individuals have rational expectations, the behavior of a 'representative household' is unrelated to the absence or presence of rational expectations on the micro level, and therefore lacks a microeconomic foundation. This premise is consistent with the problem we previously noted where arbitrageurs need to determine whether or not $(o)^t$ or $(y)^t$ has increased or decreased due to a change in economic assumptions. The Sonnenschein-Mantel-Debreu theorem

[15] "General theory is an obscure book. I am not sure that even Keynes himself knew completely what he really meant… We are in a much better position than Keynes was to figure out how the economy works… Classical economics is right in the long run… Moreover, economists today are more interested in the long-run equilibrium." Mankiw, N. Gregory (1992). *European Economic Review*, pp. 560-561.

[16] John F. Muth (1961) "Rational Expectations and the Theory of Price Movements" reprinted in *The New Classical Macroeconomics. Volume 1*. (1992): pp. 3-23 (International Library of Critical Writings in Economics, vol. 19. Aldershot, UK: Elgar.)

raises the specter that generalized assumptions about the cost-of-carry may be inconsistent with the specific operating context and microeconomic assumptions of an individual bona fide hedger.

As indicated previously, Keynes (1923, 1930) formulated his commodity hypothesis and the "theory of normal backwardation" arguing that F_t is typically less than $E(S_t)$. This is based on the two assumptions. First, market participants are risk averse (this is not universally agreed upon by academics), therefore $E(S_t) > S_0(o + r - y)^t$, which also implies that F_t is naturally $< E(S_t)$; accordingly, the futures markets are *normally* backwardated. Second, commodities, which are used for consumption or production purposes, may not be easily shorted (S_0 borrowed and sold). As a consequence, arbitrage cannot force $F_t = S_0(o + r - y)^t$; rather, it can only assure that $F_t \leq S_0(o + r - y)^t \leq E(S_t)$. Yet Keynes' theory allows for the possibility of $E(S_t) < S_0(o + r - y)^t$, which we note also implies that F_t can be $> E(S_t)$.[17] Accordingly, there are situations when Keynes' classic model acknowledges that the futures markets can exhibit contango conditions, although the term was never specifically used by Keynes in describing his theory. Nevertheless, Keynes theory of normal backwardation does not detract from the concern that the classic arbitrage model exhibits circular logic, rather the theory of congenital weakness just places certain constraints on the model.

At the time Keynes et al. offered little in terms of empirical evidence for the theory of normal backwardation: "Since the expected future spot price is not observable, the signature of normal backwardation will be the tendency of the forward price to rise (more than the opportunity costs of holding the commodity would suggest) as the delivery date approaches."[18] Such notions persist despite studies such as Allen, Cruickshank, Morkel-Kingsbury and Souness (1999) who concluded that "few of the contracts studied consistently exhibit normal backwardation while many show evidence of contango." Yet even to this day, mainstream *wisdom* endures that backwardation is the "natural" state for futures on commodities as a result of *normal* arbitrage pressures. A key reason why this belief persists, despite lack of empirical proof, is probably related to confusion surrounding the generally accepted definition of backwardation and contango in accordance with the simplified arbitrage model. It may be that institutions may have a vested interest in perpetuating such ideas.

We offer the following hypothesis as to why research continues to produce inconsistent results with respect to studies on futures market risk premia: Despite that rational expectations equilibrium is a core underlying assumption, modeling the relation between S_0 or F_t versus $E(S_t)$ intrinsically encompasses some form of circular logic, and accordingly, is subject to reflexivity.

[17] Synopsis of equations for Keynes theory sourced from Rubenstein, Mark. 2006. "A History of the Theory of Investments" Publisher: *Wiley*, ISBN-10: 0471770566.

[18] Quoted passages on Keynes theory sourced from Rubenstein, Mark. 2006. "A History of the Theory of Investments" Publisher: Wiley, ISBN-10: 0471770566.

To resolve this otherwise, requires an impracticable, quixotic control condition. Therefore, since the classic arbitrage model is *actually* reflexive, its *natural* state is disequilibrium not equilibrium, and the slightest change to any of the variables will trigger price movement in one direction or the other. This creates a feedback loop within this particular model, as well as reflective feedback within complimentary models such as our theory of "roll yield permutations" and Spurgin's (2000) "hedging response model." As an aside, there may also be biases built into the construction of continuous series of futures data, which also instigates contradictory results from empirical research.

Regardless, the classic arbitrage model, while an abstraction from reality, provides significant insight into various aspects of how the futures market operates. Further, we do not claim that backwardated or contango conditions cannot exist in the classic sense; that is, S_0 plus cost-of-carry is < $E(S_t)$, or F_t net cost-of-carry is > $E(S_t)$. In accordance with the Sonnenschein-Mantel-Debreu theorem, we theorize that individual hedgers can readily determine such conditions in relation to their specific business and economic situation at a particular point in time. Rather, it is impossible for the broad mass of market participants, specifically a "crowd" of speculators, to know perfectly whether ε equals zero [or ε = factor of 1] and $(o)^t$ or $(y)^t$ has increased or decreased due to a change in micro or macroeconomics, or whether ε is negative or positive [or ε > or < factor of 1] and an arbitrage opportunity exists. Further, analysis of market fundamentals is highly prone to subjectivity and error—this is well understood by professional traders who rely on money management techniques, and perhaps why the key to Spurgin's (2000) hedging response model is a behavioral risk management mechanism.

Simplified Arbitrage Model

The "simplified arbitrage model" is based on a conventional and mainstreamed interpretation of backwardation and contango. The model looks at the 'term structure' of the 'futures price curve,' and compares the nearby futures contract with such futures' subsequent contract months. If the nearby futures contract price is trading at a premium (higher) than the 'second nearby' or following successive delivery contracts, the market is said to be backwardated or in backwardation. If the nearby futures contract price is trading at a discount (lower) than the 'second nearby' or following successive delivery contracts, the market is said to be in contango. It is noted here that the "term structure of the futures price curve" provides the underlying rationale for an alleged 'structural risk premium' known as the 'roll return' or 'roll yield' (term used depends on the literature source).

The model focuses on the relation between present state variables only. These variables are the (i) current spot price of an asset; (ii) current nearby futures contract price; and (iii) the current second nearby and/or successive futures contract prices. Let S_0 be the current spot price of the asset; F_1 be the current futures contract price for nearby future delivery of the underlying asset, and F_2, F_3, F_4 be the current futures contract price for second nearby, third nearby and fourth nearby future delivery of the underlying asset. By not taking into account $E(S_t)$, the expected future spot price, examination of relative pricing is made uncomplicated. Based on Kaldor's (1939) ideas about 'supply-of-storage,' Working (1948) observed that since storage costs are normally higher the longer a commodity is stored, the futures price at increasingly distant delivery dates will naturally be higher than at earlier dates, and that the difference will be the cost of storage. As a consequence, the *natural* term structure of the futures price curve is contango such that $S_0 < F_1 < F_2 < F_3 < F_4$.

This seeming contradiction between normal backwardation and the theory of storage is reconciled by the classic arbitrage model in that the term structure of the futures price curve can be contango, but the underlying expected future spot price can also be backwardated in relation to each of its corresponding futures contract. Therefore, if $S_0 < F_1 < F_2 < F_3 < F_4$ is true and indicates contango, then $F_1 < E(S_1)$, $F_2 < E(S_2)$, $F_3 < E(S_3)$, and $F_4 < E(S_4)$ can also be true and is indicative of backwardation, where $E(S_1)$ is the expected future spot price for F_1 future delivery, $E(S_2)$ is the expected future spot price for F_2 future delivery, etc. Likewise the reverse scenario is possible. However, while this approach is based on probabilistic reasoning and reconciles the divergence between normal backwardation and the impact of time on storage outlays, theorists generally prefer to reference Working's (1948) research on the relationship between inventories and carrying charges in order to provide an economic rationale for resolving this seeming contradiction. Working concluded that (a) 'inverse carrying charges' are a reliable indicator of scarcity and that (b) during times of scarcity, Kaldor's (1939) convenience yield becomes sufficiently large as to overwhelm a commodity's storage and financing costs, leading to a negative price of storage.

As initially put forward, the simplified arbitrage model describes the futures price curve characteristics from which the supposed structural risk premium known as the roll return is derived. As indicated by our exposé of the classic arbitrage model, for the economic relationship between a futures contract and the underlying cash commodity to exist, it is necessary that the underlying be deliverable against that futures position (either in the form of physical delivery or cash settlement). Interestingly, across all futures markets, less than one percent of open futures positions are settled by delivery.[19] However, the fact that delivery may be made means that the price of a futures contract must relate realistically to the price of the underlying commodity. This relationship is facilitated

[19] Source is testimony by Reuben Jeffery III, Chairman U.S. Commodity Futures Trading Commission on November 2, 2005 before the Committee on Energy and Commerce United States House of Representatives.

through arbitrage, ensuring convergence during delivery month, which is induced by the classic arbitrage model. However, since speculators rarely take or make delivery of the underlying commodity, and thus rarely hold a futures contract through delivery settlement, a speculator's exposure to the commodity futures price eventually involves closing out the held contract and "rolling" into a successive futures position with a later expiry date. This process, known as 'rolling the futures contract forward,' is said to potentially generate an 'excess return' which is either positive or negative depending on whether the speculator is long or short, and whether the futures price curve is backwardated or contango within the definition of the simplified arbitrage model.

This is the rationale which underlies the proposition that commodity futures investment encompasses returns from three sources: (i) the risk-free return from investment in collateral set aside for *de minimis* margin (such good faith deposits are often placed in US Treasury bills); (ii) the change in futures prices from the point at which a position is initiated until it is either liquidated (in the case of longs) or covered (in the case of shorts) [note: in keeping with convention, we shall refer to this change in prices as 'spot return']; and (iii) the roll return or roll yield, which can be either positive or negative, and results from replacing an expiring contract with a further out contract in order to avoid physical delivery, yet maintain a long or short exposure in the markets market.

The convention for calculating spot returns, excess returns and roll returns, as explained by Shimko and Masters (1994), is as follows: let F_1^t equal the nearby futures contract price at the current point in time t, let F_1^{t+1} equal the nearby futures contract price at a distant point in time $t+1$, let F_2^t equal the second nearby futures contract price at the current point in time t, let F_2^{t+1} equal the second nearby futures contract price at a distant point in time $t+1$. Given these variables, excess return = $(F_1^{t+1} - F_2^t) / F_2^t$, spot return = $(F_1^{t+1} - F_1^t) / F_1^t$, and the roll return = $((F_1^{t+1} - F_2^t) / F_2^t) - ((F_1^{t+1} - F_1^t) / F_1^t)$. As an example, let F_1^t = \$100, let F_1^{t+1} = \$120, let F_2^t = \$90, and let F_2^{t+1} = \$100.

Point in time	t	$t+1$	$t+2$
First nearby	F_1^t = \$100	F_1^{t+1} = \$120	N/A
Second nearby	F_2^t = \$90	F_2^{t+1} = \$100	N/A

As a result, excess return equals (\$120 – \$90) / \$90 or 33%, spot return equals (\$120 – \$100) / \$100 or 20%, and the "arithmetic" roll return is equal to 33% – 20% = 13%.[20] The roll return essentially propositions a speculative play at capturing convenience yield without paying for the outlay of storage costs. However, this trade is not without speculative risk, regardless of any roll yield benefits.

[20] Formulas and sample calculation sourced from Till, Hilary. 2007. "Part I of A Long-Term Perspective on Commodity Futures Returns: Review of the Historical Literature," Intelligent Commodity Investing, (Till, and Eagleeye, Ed.), Published by Risk Books, a Division of Incisive Financial Publishing, Ltd., pp. 39-82. [Note that while values are the same, variables are labeled using our conventions, not those in Till's exposé.]

First, we must point out an issue with the above math in regards to the convention for calculating excess returns, and as a consequence the supposed roll return. While the formula above is mathematically possible and calculates a yield that approximates what a speculator *might* capture when rolling the contract forward, it is physically impossible in the *real world* to roll from F_1^{t+1} into F_2^{t}, since the former is a price from point in time $t+1$, and the latter is a price from point in time t. Accordingly, the calculation for excess return is an accounting fiction. Further, the use of F_2^{t} as the denominator in the excess return calculation, $(F_1^{t+1} - F_2^{t}) / F_2^{t}$, results in a performance skew over time. Specifically, using F_2^{t} (i.e., $90) as the denominator rather than F_1^{t} (i.e., $100) as the denominator is akin to collateralizing the trade at 90 cents on the dollar rather than 100 cents on the dollar. While this is technically possible in futures trading due to notional funding, performance over time theoretically results in leveraged returns. We therefore surmise that empirical studies which calculate the roll return using Shimko and Masters' convention reflect flawed conclusions.

What actually happens in the *real world* is as follows: To begin, we shall include an additional variable F_2^{t+2}, where the second nearby futures contract is also priced at point in time $t+2$. In our revised *real world* example, a speculator goes long the contract at F_1^{t} and liquidates the contract at F_1^{t+1}, at which time he/she rolls into the second nearby futures contract at F_2^{t+1}, and then subsequently closes out the trade by liquidating the contract at F_2^{t+2}. Let's assume that F_2^{t+2} = $100 but all other variables are the same as in Till's (2007) original example given in the above paragraph:

Point in time	t	$t+1$	$t+2$
First nearby	F_1^{t} = $100	F_1^{t+1} = $120	N/A
Second nearby	F_2^{t} = $90	F_2^{t+1} = $100	F_2^{t+2} = $100
Alt. second nearby	F_2^{t} = $90	F_2^{t+1}=$95 / $E(S_2)$=$100	F_2^{t+2} = $100

The formula we propose, $[\{1 + ((F_1^{t+1} - F_1^{t}) / F_1^{t})\} \cdot \{1 + ((F_2^{t+2} - F_2^{t+1}) / F_2^{t+1})\} - 1]$, correctly calculates in this example a profit of $20 and a geometric return of 20% to the speculator. At the same time, there is no obvious roll return across the entire trade. However, if F_2^{t+1} had alternatively been $95 rather than $100, one could hypothetically argue that there a $5 roll return resulted from the $(F_2^{t+2} - F_2^{t+1})$ trade. This argument, that there was a roll yield benefit resulting from rolling the contract forward, is rooted on the assumption that the F_2^{t+2} liquidation price of $100 at point in time $t+2$ is equivalent to $E(S_2)$ at point in time $t+1$, which theoretically reflects a value of $S_0(o \pm y \pm \varepsilon)^{t+2}$. Otherwise the $5 difference between F_2^{t+2} and F_2^{t+1} could be accountable to just a change in the spot price of F_2 from $t+1$ to $t+2$. Here again, one must return to the classic arbitrage model in order to calculate roll yield. At minimum this approach reconciles the math with the *real world*.

Second, we posit that roll return is nothing more than a calendar spread trade. Till (2007) incorrectly states that "one cannot invest and receive the spot return separate from the roll return; and, correspondingly, one cannot invest solely to receive the roll return, but not the spot return." [We admit this may only be semantics, in which Till points out, similar to our point above about the fictional math versus the *real world*, that "the convention of separating out futures-only returns into spot return and roll return is solely for performance-attribution purposes."] Perhaps this is a syntactical issue, but as we have defined above, and Shimko and Masters' (1994) formula apparently validates, the spot return is equal to the change in price of the futures contract. Therefore…

Since a calendar spread trade is a relative risk trade, one can simultaneously sell the nearby futures contract and buy the second nearby futures contract if the term structure is considered backwardated; or vice versa, buy the nearby futures contract and sell the second nearby futures contract if the term structure is considered contango. Accordingly, one could develop a trading program that isolates the roll yield phenomena in this way, and without concern whether 'normal backwardation' is built into the futures price curve, since relative risk trades allow a speculator to take advantage of supposed backwardation or contango conditions either way. In fact, de Roon, van den Goorbergh and Nijman (2004) found that net hedging pressure was exploitable *only* through futures calendar spreads rather than outright futures contracts. (Admittedly, we may be mixing the roll yield hypothesis with Cootner's (1967) net-hedging pressure hypothesis, but this goes to further our evolving argument that the various commodity pricing models do not operate to the exclusion of each other, but are related.) With respect to isolating spot return, this can be facilitated by either trading on a short term basis without the need to roll contracts, or by creating what is now called a 'portable alpha' construct. This is hypothetically possible if one trading program trades calendar spreads in opposite direction of a long-only (or vice versa, short-only) trading program.

Our third point of consideration is that widespread application of calendar spread trading and "next generation" long-short commodity index investing (which in part developed from within the context of the simplified arbitrage model) could hypothetically result in a persistent *flat* term structure. In theory, a flat futures price curve would represent equilibrium in the model, which is why the simplified arbitrage model is fallacious. Based on commonly-held assumptions derived from this model, speculators should theoretically only sell premium (i.e., contango) futures contracts and only purchase discount (i.e., backwardated) futures contracts. With respect to the structural risk premium that is supposedly derived from the roll yield, *real-life* speculators have already developed trading programs with the idea of getting in front of the flow-of-money of publicly disclosed roll dates of passive institutional commodity funds. Therefore, in theory, the so-called roll return is reduced if not eliminated by the very institutions (i.e., speculators) that are trying to capture it. This explains why institutional sponsors of passive long-only commodity index products have become so concerned with term structure. Lewis (2007) writes that "the traditional approach to rolling commodity index futures on a predefined monthly schedule is in need of reform to address the implications that unstable term structures imply for the roll return within a commodity index."

Fourth, we also need to point out that the simplified arbitrage model and ensuing roll returns and calendar spreads are nothing more than an imperfect hedge. If the speculators in our examples were bona fide hedgers who engaged in arbitrage, then such arbitrageurs would be able to perfectly hedge the relative change in the futures contract price against embedded carrying charges by maintaining the position to physical or cash settlement of the futures contract. In reality, many commodity futures contracts enter into a delivery notice period prior to the contract's expiration date, whereby a holder of a long or short futures contract is exposed to assignment to take delivery or make delivery of the underlying commodity. For this reason, only well capitalized speculators or bona fide hedgers can actually enter into this period without delivery risk. Hypothetically, this may be the exact period when Keynes' "signature of normal backwardation" is most apparent. As we stated previously, the roll return is essentially a speculative play at trying to capture the convenience yield without paying for the outlay of storage costs. However, it needs to be acknowledged that establishing outright positions (versus relative positions) exposes the speculator to a more significant risk of changes in price (i.e., spot return). Even accepting all of the theory and research intended to prove the existence of the roll return, such minute yield, either beneficial or detrimental, arguably is only relevant within the context of long-term bull or flat market cycles assuming markets are normally backwardated. It may be presently fashionable to be in commodities now, but where was everyone in the late 1990s? As many studies reiterate, roll return/yield as a structural risk premium is only relevant over the very long-term, and short-term analysis is likely to exhibit material sample period bias.

Roll Yield Permutations

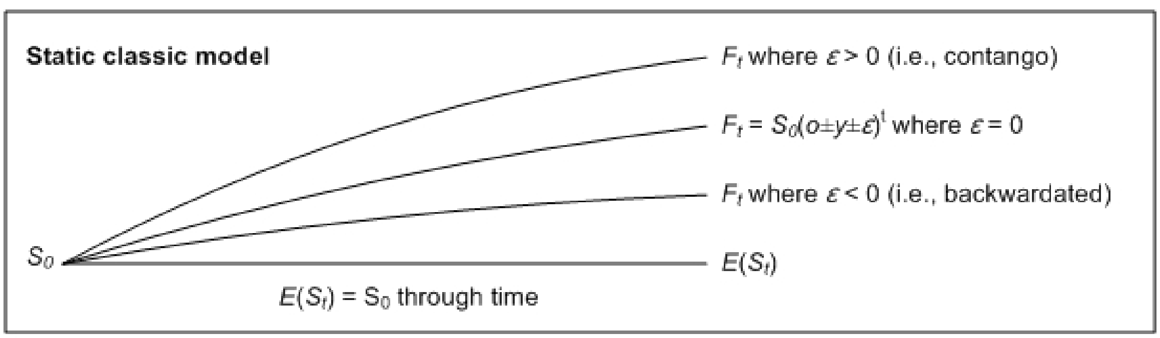
Our theory of "roll yield permutations" combines characteristics of both the classic arbitrage model and the simplified arbitrage model. This idea was touched upon when we showed that it was possible for the term structure of the futures price curve to be contango within the context of the simplified arbitrage model, while at the same time the expected future spot price can be backwardated in relation to the same term structure within the context of the classic arbitrage model. At the expense of repeating ourselves, we will start this section by repeating some previously covered concepts.

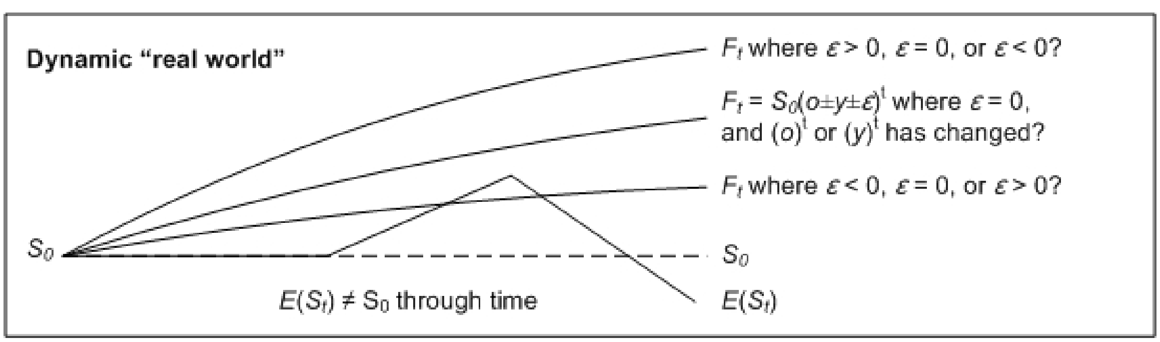
Let S_0 be the current spot price of the asset, and F_0 be the current futures contract price for an illiquid futures contract trading after first notice within the delivery period but prior to last physical delivery date and contract expiry date; let F_1 be the current futures contract price for the liquid nearby future delivery of the underlying asset, and F_2, F_3, F_4 be the current futures contract price for second nearby, third nearby and fourth nearby future delivery of the underlying asset.

Additionally, let $E(S_1)$ be the expected future spot price for F_1 future delivery, $E(S_2)$ be the expected future spot price for F_2 future delivery, $E(S_3)$ be the expected future spot price for F_3 future delivery, and $E(S_4)$ be the expected future spot price for F_4 future delivery. We shall assume that each of F_0, F_1, F_2, F_3, and F_4 futures contracts are physically settled in the current year December, and following year March, June, September and December, respectively.

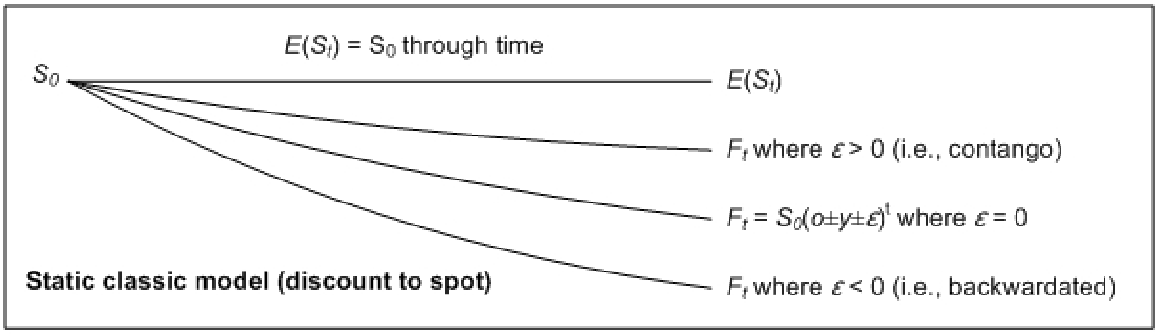
As previously discussed, the theory of storage implies that the *natural* term structure of the futures price curve is contango such that $S_0 < F_1 < F_2 < F_3 < F_4$. This seeming contradiction between normal backwardation and the theory of storage is reconciled by the classic arbitrage model in that the term structure of the futures price curve can be contango, but the underlying expected future spot price can also be backwardated in relation to its corresponding futures contract. Therefore, if $S_0 < F_1 < F_2 < F_3 < F_4$ is true and indicates contango, then $F_1 < E(S_1)$, $F_2 < E(S_2)$, $F_3 < E(S_3)$, and $F_4 < E(S_4)$ can also be true and is indicative of backwardation. Likewise the reverse scenario is possible with the futures price curve backwardated, but the underlying expected future spot prices contango in relation to each of the corresponding futures contracts. The following two graphics extrapolate on our previous discussion and illustrate the range of possibilities, as well as the potential complexities that occur when the simple arbitrage model is overlaid with the classic.

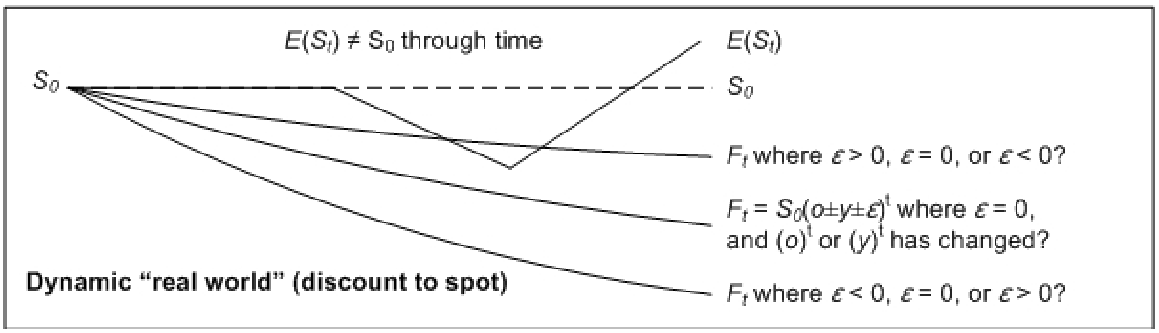
This first graphic was presented along with the classic arbitrage model, and stresses Working's (1948) empirical observations about the relationship between futures prices and storage costs.





The second graphic on the following page, extends the model's reflexivity qualities, illustrating how the variables operate in an environment where the futures price curve reflects backwardation.





Before continuing, we will again review a typical scenario which supposedly illustrates the potential for capturing the roll return (or risk premia) from rolling the contract forward. (For testing, scenarios should be applied to each of the permutations describe below, revealing the richness of complexities resulting from our roll yield permutations hypothesis.) Let F_1^t be the nearby futures contract price at the current point in time t, let F_1^{t+1} be the nearby futures contract price at a further-out point in time $t+1$, let F_2^t be the second nearby futures contract price at the current point in time t, let F_2^{t+1} be the second nearby futures contract price at a further-out point in time $t+1$, and let F_2^{t+2} be the second nearby futures contract price at a distant point in time $t+2$. For our first example, let F_1^t = \$100, let F_1^{t+1} = \$120, let F_2^t = \$90, let F_2^{t+1} = \$100, and let F_2^{t+2} = \$100. In this example, a speculator goes long the futures contract at F_1^t and liquidates the contract at F_1^{t+1}, at which time he/she rolls into the second nearby futures contract at F_2^{t+1}, and then subsequently closes out the trade liquidating the contract at F_2^{t+2}. As a consequence, the trade realizes for the speculator the following profit: $((F_1^{t+1} - F_1^t) + (F_2^{t+2} - F_2^{t+1})) / F_1^t$, which results in ((\$120 – \$100) + (\$100 – \$100)) / \$100 = \$20 / \$100 or 20% return on the original investment. Consequently, there is apparently a \$0 roll return after the complete sequence. This is a *seemingly* straightforward conclusion.

For the second example, let's assume that all the variables remain the same except F_2^{t+1}, which is assumed to be \$95 rather than \$100 as in the prior example. As a result of this change, the trade now realizes for the speculator the following profit: ((\$120 – \$100) + (\$100 – \$95)) / \$100 = \$25 / \$100 or 25% return on the original investment. Interestingly, whether or not there a roll yield benefit resulted from rolling the contract forward is based on a supposition related to the classic arbitrage model. The \$5 difference between F_2^{t+2} and F_2^{t+1} is evidently accountable to just a change in the spot price of F_2 from $t+1$ to $t+2$. However, one could hypothetically argue that there is a \$5 roll return resulting from the $(F_2^{t+2} - F_2^{t+1})$ trade. This line of reasoning, that a roll return ensued from rolling the contract forward, is based on the assumption that the F_2^{t+2} liquidation price of \$100 at point in time $t+2$ is equivalent to $E(S_2)$ at point in time $t+1$, which theoretically reflects a value of $S_0(o \pm y \pm \varepsilon)^{t+2}$. In effect, the classic arbitrage model reconciles the math with the idea of a structural risk premium related to rolling contracts. But this conclusion is solely based on an assumption that we know the value of $E(S_2)$ at point in time $t+1$, which as stated previously is an unknown.

It therefore logically follows that the accepted convention for calculating the roll yield, with the roll return formulated to equal $((F_1^{t+1} - F_2^t) / F_2^t) - ((F_1^{t+1} - F_1^t) / F_1^t)$, represents a rough approximation at best, or bad assumption at worst, for average pricing of backwardation or contango conditions. This is because rolling roll from F_1^{t+1} into F_2^t is physically impossible in the *real world*, since the former is a price from point in time $t+1$, and the latter is a price from point in time t. Accordingly, the calculation for excess return is an accounting fiction. This is supposedly explained by Shimko and Masters (1994), who state that "the convention in calculating excess returns is to treat the futures investment as being fully collateralized based on the second nearby price." We, on the other hand concur with Lavoie (1992b) who warns us: "Axioms are chosen, not for their likelihood, but for their ability to allow the existence of equilibrium or its uniqueness. Neo-Walrasians describe the world as it should be rather than as it is." Stated otherwise, in our pursuit for understanding the *real world* through hypothesis testing, we must not fall into the trap of accepting mainstream models because it is mathematically possible. Not questioning a models' logic and the formulae upon which widely-held conclusions and misunderstandings are perpetuated, simply because it may lead to the whole system crumbling, is intellectually dishonest (as well as only gainful in the short run). For this reason we put forth our theory of roll yield permutations which demonstrates how difficult it is to accurately test for roll returns empirically, and questions testing methods as well as the persistence of such risk premia.

It should be noted, that we do not query the veracity of the roll return lightly, and thus offer the following comparison to the well-vetted Black-Scholes options pricing model. One way to capture risk premium is by selling out-of-the-money naked options (i.e., short gamma option strategy). In such a strategy, since the strike price is known and time decay (i.e., theta) depreciates the value of the option over time assuming that the underlying asset remains out-of-the-money, a speculator has a way of gauging the risk of the trade, and knowing at any point in time whether or not the trade is working in his/her favor based on changes in the underlying asset price relative to such *hard-coded* option strike price. The roll return/yield hypothesis, which is based on the simplified arbitrage model of backwardation and contango, implies a similar construct to the above short naked option strategy. The risk premium, in the case of roll returns, is assumed to equal carrying charge including convenience yield, and which through time also decays as the contract approaches expiration and physical or cash settlement. In contrast, the material difference between the models, however, is that the expected future spot price, which we note is analogous to the option strike, is not locked into a specific static price objective. The expected future spot price is, in fact, a moving target valued in accordance with its own particular fundamental supply-demand dynamics. Worse, it is by definition an unknown that can only be theoretically approximated by vicarious interpolation between the spot price and the futures price curve, as previously discussed in detail. Consequently, the convention for calculating roll returns is an equation not without inherent relativity issues due to the fact that its "strike price," the expected futures spot price, is a moving target and an unknown until conversion into the spot price.

Having reviewed pertinent background information, and building upon the models, formulae and variables so far established, we can now focus on the "roll yield permutations" model. First, let us assume that there are three types of market conditions: (i) a hypothetical flat market where there is absolutely no price change for any variable, (ii) a bull market where variables generally trend upwards, and (iii) a bear market where variables generally trend downwards. Again, let S_0 be the current spot price of the asset, and F_0 be the current futures contract price for an illiquid futures contract trading after first notice day and within the delivery period, but prior to the last physical delivery date and contract expiration; let F_1 be the current futures contract price for the liquid nearby future delivery of the underlying asset, and F_2, F_3, F_4 be the current futures contract price for second nearby, third nearby and fourth nearby future delivery of the underlying asset. Additionally, let $E(S_1)$ be the expected future spot price for F_1 future delivery, $E(S_2)$ be the expected future spot price for F_2 future delivery, $E(S_3)$ be the expected future spot price for F_3 future delivery, and $E(S_4)$ be the expected future spot price for F_4 future delivery. Further, we shall assume that each of F_0, F_1, F_2, F_3, and F_4 futures contracts are physically settled in the current year December, and the following year March, June, September and December, respectively.

Permutation Level 1: Assuming one week prior to contract expiration, after first notice day and within the delivery period, but prior to the last physical delivery date and contract expiration, there are three likely scenarios with respect to the relationship between S_0 and F_0. Either: $F_0 < S_0$, or $F_0 = S_0$, or $F_0 > S_0$, with convergence occurring at the time of physical settlement, such that $S_0 = F_0$ on the contract settlement date. This set of permutations relates to the one percent of open interest futures positions that are settled through delivery, either by bona fide hedgers or by institutional arbitrageurs who still hold the contract into this period of the contract's life cycle. At this juncture, most speculators have rolled forward and are trading the next liquid futures contract month.

Permutation Level 2: The next set of permutations revolves around the relationship between S_0 and F_1. For this set of permutations we extend the three basic possibilities described in Permutation Level 1 to encompass bullish market conditions, flat market conditions and bearish market conditions. Hence, we note that there are three sets of permutations for each type of market condition. Assuming bullish market conditions, either: $F_1 < S_0$, or $F_1 = S_0$, or $F_1 > S_0$; assuming flat market conditions, either: $F_1 < S_0$, or $F_1 = S_0$, or $F_1 > S_0$; and assuming bearish market conditions, either: $F_1 < S_0$, or $F_1 = S_0$, or $F_1 > S_0$. However, for each of these permutations, given the potential for either backwardation, contango or equilibrium market conditions as such relate to the classic arbitrage model, it is necessary to relate the F_1 future delivery to its corresponding $E(S_1)$ (i.e., expected future spot price). As a result, we can extend the model to include nine possible permutations for each type of market condition. Assuming bullish conditions and we know the value of $E(S_1)$, then either:

$F_1 < S_0$, where $F_1 < E(S_1)$
$F_1 < S_0$, where $F_1 = E(S_1)$
$F_1 < S_0$, where $F_1 > E(S_1)$
$F_1 = S_0$, where $F_1 < E(S_1)$
$F_1 = S_0$, where $F_1 = E(S_1)$
$F_1 = S_0$, where $F_1 > E(S_1)$
$F_1 > S_0$, where $F_1 < E(S_1)$
$F_1 > S_0$, where $F_1 = E(S_1)$
$F_1 > S_0$, where $F_1 > E(S_1)$

As mentioned above, these nine permutations can be applied to flat market conditions and bearish market conditions too. The significance of the underlying market conditions relates to the benefit or detriment the roll yield theoretically provides under either bullish, flat and bearish market scenarios. For example, assuming an established long position, each roll into the forward contract month could result in one of the following theoretical scenarios/outcomes: (1) roll during a bull market in backwardation results in a yield benefit and a positive price change; (2) roll during a bear market in backwardation results in a yield benefit but a negative price change; (3) roll during a flat market but expected future spot market is backwardated results in a yield benefit and zero price change; (4) roll during a neutral market which is neither backwardated or contango results in zero yield benefit and zero price change; (5) roll during a flat market but expected spot market is contango results in a yield detriment and zero price change; (6) roll during a bull market in contango results in a yield detriment but a positive price change; and (7) roll during a bear market in contango results in a yield detriment and a negative price change. Likewise, assuming an established short position, the opposite of these seven theoretical scenarios/outcomes could result.

Additionally, markets do not go straight up in bull markets or straight down in bear markets, and flat markets could go both up then down, down then up, sideways, or any combination thereof. Therefore, assuming that each of F_0, F_1, F_2, F_3, and F_4 futures contracts are physically settled in the current year December, and following year March, June, September and December, respectively; and further, we assume four rolls are transacted in the following year: December to March, March to June, June to September, and September to December; then during each quarterly roll, twelve separate market patterns could theoretically occur: (i) bull, bull, bull, bull; (ii) bull, bear, bull, bull; (iii) bull, bull, bear, bull; (iv) bull, bear, bear, bull; (v) bull, bull, bear, bear; (vi) bull, bear, bull, bear; (vii) bear, bear, bear, bear; (viii) bear, bull, bear, bear; (ix) bear, bear, bull, bear; (x) bear, bull, bull, bear; (xi) bear, bear, bull, bull; and (xii) bear, bull, bear, bull. In order to appreciate the full complexity of permutations, the beneficial or detrimental roll yield scenarios should be overlaid upon each of the quarterly rolls, within the twelve separate market pattern scenarios described above.

It should be noted that these nine permutations can be extended to include the relationships between S_0 and F_2, S_0 and F_3, or S_0 and F_4, resulting in twenty-seven additional permutations assuming the following four potential roll scenarios in any one year: December to March, December to June, December to September, and December to December. It is also noted that these permutations in the *real world* are a difficult set of relationships for the average speculator to arbitrage since these permutations involve the cash spot market (i.e., S_0) and thus require actual outlay for storage costs.

<u>Permutation Level 3</u>: The next set of permutations builds upon ideas presented in Permutation Level 1 and 2, but extends established concepts around the relationship between F_1 and F_2 (and by extrapolation, the relationships between F_1 and F_2, F_3, F_4; and F_2 and F_3, F_4; and F_3 and F_4). Therefore, if one assumes bullish market conditions, then either: $F_1 > F_2$, or $F_1 = F_2$, or $F_1 < F_2$; assuming flat market conditions, then either: $F_1 > F_2$, or $F_1 = F_2$, or $F_1 < F_2$; and assuming bearish market conditions, then either: $F_1 > F_2$, or $F_1 = F_2$, or $F_1 < F_2$. However, for

each of these permutations, given the potential for either backwardation, contango or equilibrium market conditions as such relate to the classic arbitrage model, it is necessary to relate the F_1 future delivery to its corresponding $E(S_1)$, and also relate the F_2 future delivery to its corresponding $E(S_2)$. As a result, the model is extended to include twenty-seven possible permutations for each type of market condition. Therefore, for each underlying market condition, and assuming we know the value of $E(S_1)$ as well as $E(S_2)$, then either:

$F_1 > F_2$, where $F_1 > E(S_1)$ and $F_2 > E(S_2)$
$F_1 > F_2$, where $F_1 = E(S_1)$ and $F_2 > E(S_2)$
$F_1 > F_2$, where $F_1 < E(S_1)$ and $F_2 > E(S_2)$
$F_1 > F_2$, where $F_1 > E(S_1)$ and $F_2 = E(S_2)$
$F_1 > F_2$, where $F_1 = E(S_1)$ and $F_2 = E(S_2)$
$F_1 > F_2$, where $F_1 < E(S_1)$ and $F_2 = E(S_2)$
$F_1 > F_2$, where $F_1 > E(S_1)$ and $F_2 < E(S_2)$
$F_1 > F_2$, where $F_1 = E(S_1)$ and $F_2 < E(S_2)$
$F_1 > F_2$, where $F_1 < E(S_1)$ and $F_2 < E(S_2)$

$F_1 = F_2$, where $F_1 > E(S_1)$ and $F_2 > E(S_2)$
$F_1 = F_2$, where $F_1 = E(S_1)$ and $F_2 > E(S_2)$
$F_1 = F_2$, where $F_1 < E(S_1)$ and $F_2 > E(S_2)$
$F_1 = F_2$, where $F_1 > E(S_1)$ and $F_2 = E(S_2)$
$F_1 = F_2$, where $F_1 = E(S_1)$ and $F_2 = E(S_2)$
$F_1 = F_2$, where $F_1 < E(S_1)$ and $F_2 = E(S_2)$
$F_1 = F_2$, where $F_1 > E(S_1)$ and $F_2 < E(S_2)$
$F_1 = F_2$, where $F_1 = E(S_1)$ and $F_2 < E(S_2)$
$F_1 = F_2$, where $F_1 < E(S_1)$ and $F_2 < E(S_2)$

$F_1 < F_2$, where $F_1 > E(S_1)$ and $F_2 > E(S_2)$
$F_1 < F_2$, where $F_1 = E(S_1)$ and $F_2 > E(S_2)$
$F_1 < F_2$, where $F_1 < E(S_1)$ and $F_2 > E(S_2)$
$F_1 < F_2$, where $F_1 > E(S_1)$ and $F_2 = E(S_2)$
$F_1 < F_2$, where $F_1 = E(S_1)$ and $F_2 = E(S_2)$
$F_1 < F_2$, where $F_1 < E(S_1)$ and $F_2 = E(S_2)$
$F_1 < F_2$, where $F_1 > E(S_1)$ and $F_2 < E(S_2)$
$F_1 < F_2$, where $F_1 = E(S_1)$ and $F_2 < E(S_2)$
$F_1 < F_2$, where $F_1 < E(S_1)$ and $F_2 < E(S_2)$

As can be inferred by the variety of roll yield permutations above, the number of potential manifestations can be overwhelming when one also considers variations as a result of timing rolls.

Economic modeling is often premised on the idea of distilling complex data sets into axioms that can be robustly applied to the intricacies of the *real world*. However, the purpose of our roll yield permutations model is to cause the opposite reasoning: take self-evident precepts about the commodities futures market and reveal how truly complex the trade decision-making process is. We do not portend that these permutations allow for "natural" economic constraints or elucidate the underlying dynamics of the arbitrageur, hedger, speculator paradigm in the legacy of Keynes, Hicks, Kaldor, Working, Brennan, Cootner, etc. Rather, these permutations demonstrate the range of possibilities when one clarifies the simplified arbitrage model by incorporating the pivotal relationship between F_t and its corresponding $E(S_t)$, as elucidated by the classic arbitrage model. Accordingly, we admit that if F_2 is backwardated relative to F_1 [i.e., $F_1 > F_2$], then it is a likely indication that $E(S_2)$ is also backwardated relative to $E(S_1)$ [i.e., $E(S_1) > E(S_2)$], but this may not always be the case, and researchers and practitioners should not presume as much.

Permutation Hypothesis: Our hypothesis for the classic arbitrage model propositions that the model contains circular logic, and as a consequence is subject to reflexivity. If such model is in fact a reflexive model, we propose that its *natural* state is disequilibrium, not equilibrium. We now extend this hypothesis to suggest that the term structure of the futures price curve, while indicative of a potential roll return benefit or detriment, in fact implies a complex and reflexive series of roll yield permutations which amplifies into an exponential number of potential market scenarios. Further, both models do not operate to the exclusion of the other, nor exclusively from each other; rather, such models are inter-related and each reflect certain aspects and dynamics within the total futures market paradigm. Additionally, these models interact through reflective feedback between each other, as well as internal feedback within themselves. Hence, we posit that these models in fact support a 'post-Keynesian' view that the world is messy and uncertain. In the words of the Keynes (1937) himself, "The calculus of probability was supposed to be capable of reducing uncertainty to the same calculable status as that of certainty itself. This false rationalization follows the lines of the Benthamite calculus. The hypothesis of a calculable future leads to a wrong interpretation of the *principles of behavior* which the need for action compels us to adopt."

We suggest that this facet of "principles of behavior" in commodity pricing theory is addressed by Spurgin's (2000) hedging response model. The following section describes his model which we infer actually represents a behavioral risk management mechanism. In our opinion, the hedging response functions at the core of the model, augments the behavioral implications of our evolving hypothesis: that the classic and simplified arbitrage models, as well as our roll yield permutations theorem, supports the idea of market disequilibrium and social reflexivity, despite that neoclassical models find their basis in rational expectations and the efficient market hypothesis.

Hedging Response Model

The "hedging response model" described in this section is sourced from Dr. Richard Spurgin's *rough draft article* "Some Thoughts on the Source of Return to Managed Futures" (2000). We discuss it here in order to further support our thesis that the *natural* state of the commodity futures market is one of disequilibrium and social reflexivity. Please note that we have taken the liberty to reorganize and re-present Spurgin's model in a way which we hope is succinct and clear. We have added some of our own thoughts along the way, but admittedly, the wording in much of this section is taken verbatim from Spurgin's original source article.

Spurgin states that the purpose of his article is to "frame the debate in economic terms as opposed to statistical or behavioral terms.¶ The statistical argument involves pointing to the twenty-year track record of managed futures and arguing that returns have been large enough for long enough that one cannot plausibly argue that there is no source of return. However, while the statistical argument can help support an argument for managed futures, it cannot stand by itself as a rationale for the strategy. One can always argue that the past twenty years are an anomaly, and since futures trading is a zero sum game, positive returns cannot persist.¶ The behavioral argument simply points out that since markets trend, trend-following strategies will be successful over time. This rational may help explain why some investors in futures markets make money. But it can't explain why so many are willing to lose, or why it is reasonable to expect them to continue losing... The typical [speculator] doesn't care [who is] on the other side of the trade… As long as the strategy produces returns, the source of the money is irrelevant. However, the question is very relevant to [institutions]… If there is no source of return, the institution's success depends [on] being able to identify which traders will succeed and which will not. Few investors want to play that game."

The "economic terms" upon which Spurgin frames the debate relates to three of the themes influencing commodity pricing theory: (1) the insurance role of commodity futures contracts which emphasizes the role of the speculator; plus to a lesser degree, (2) the theory of storage, which emphasizes the behavior of the inventory holder and commercial hedger; but mainly (3) the net-hedging-pressure hypothesis, which encompasses the behavior of both classes of participant. Spurgin goes on to state that his article's primary objective "is to describe a set of conditions that must be present in a futures market in order to be able to argue that economic profits might exist. Another objective is to understand the set of conditions that would result in trend-following strategies being the best approach to extracting these returns." Spurgin astutely observes that even though "speculative/investment capital in futures markets is typically employed in trend-following strategies" and "models are not doing a good job of capturing the dynamics of the relationship between hedgers and speculators," studies on "the conditions necessary for trend-following strategies to extract these returns is a relatively unexplored topic. In fact, most models of futures speculation imply responsive strategies for speculators—buying futures after a decline below fair value[21] and selling after prices have risen above fair value."[22] We agree on all of Spurgin's points above except one, the article's thesis may be built on the foundation of three commodity pricing theories, but the result of such "economic terms" in fact elicits a behavioral risk management mechanism—that is, the hedging response model actually implies social reflexivity.

As an aside, it should be noted that the hedging pressure hypothesis has recently come under attack. Gorton, Hayashi and Rouwenhorst (2007) "reject the hedging pressure hypothesis as an alternative explanation for the variation of risk premiums" based on their empirical research using data obtained from the Report of Traders released by the CFTC. Their research shows that "positions of traders are contemporaneously correlated with inventories and futures prices;" however, they "find no evidence that these positions are correlated with ex-ante risk premiums of commodity futures." Spurgin (2000) readily anticipates this erroneous conclusion seven years earlier in his fourth listed condition for the hedging response model which requires a "well-developed arbitrage relationship between the underlying asset and the futures market… The fourth condition insures [sic] that the net hedging position in the market will be efficiently transmitted to the futures exchange, so speculators do not need to trade in off-exchange risk management vehicles such as forwards, swaps, and options, in order to purchase the excess risk. The role of arbitrageurs may explain why this concept has been difficult to directly test using the most obvious source of data—CFTC *commitments of traders* data. Arb traders may hold a sizeable portion of the open interest in futures, transferring hedging exposure from OTC to futures. If this is the case, commitments of traders data will not capture the full extent of hedging exposure." *But we digress…*

The hedging response model assumes there are only four types of participants in the futures market for a given product: (i) short hedgers; (ii) long hedgers; (iii) arbitrageurs; and (iv) speculators. Short hedgers are sellers (i.e., producers or distributors) who are net long the underlying cash commodity and use the futures market to fix the sale price. Long hedgers are buyers (i.e., consumers or distributors) who are net short the underlying cash commodity and use the market to fix the purchase price. Arbitrageurs are traders who make relative value trades in the cash, futures and forward markets. Speculators are traders/investors who take directional positions

[21] How to define fair value is the "million dollar question." As we have thoroughly analyzed with respect to the classic arbitrage model and the simplified arbitrage model, although such models are premised on the notion of equilibrium (i.e., defining fair value), they in fact exhibit circular logic and imply disequilibrium.

[22] Spurgin's description of a "responsive strategy" corresponds to what is called a "counter-trend strategy."

purely for potential profit. For simplicity, the model is only concerned about the net position held by each participant group: short hedgers are always net short the futures market, and long hedgers are always net long the futures market. Additionally, arbitrageurs are assumed to have zero net exposure to the underlying asset—their objective is to survey the various hedging vehicles available for the asset and take relative value positions between these vehicles. Speculators are assumed to "hold the difference between the long hedger, short hedger and arbitrageur positions."

The model's starting assumption is that the futures market is *naturally* symmetric or a zero sum game (sans transaction costs). Since arbitrageurs are assumed within the model to only engage in relative value trades and therefore have a zero net exposure, their presence in the market is also symmetric. With respect to hedgers, if hedging demand is symmetric, and both short and long hedgers have the same needs regardless of the price of the underlying, then the net hedging position will always be in balance, and speculative capital will not have an expected return. However, this equilibrium can be placed out of balance whenever hedgers on one side respond differently to changes in price versus hedgers on the opposite side. This creates a "natural" demand for speculative capital to bridge the gap, thereby encouraging the flow of capital into net speculative positions. As to speculators versus speculators, the situation is more subtle and can result in asymmetric winners versus losers for the short term, but in the long term the outcome is symmetric.

For succinctness, we shall generally abbreviate the following agents acting within the model: let (H_S) represent short hedgers (i.e., producers); let (H_L) represent long hedgers (i.e., consumers); let (A_Δ) represent speculators; and let (A_Γ) represent arbitrageurs. In addition, we shall abbreviate the following variables: let S_0 be the current spot price of the asset; F_t be the current price for future delivery of the underlying asset, and $E(S_t)$ be the expected spot price of the underlying asset on the delivery date. It is also noted that S_0 is a known variable equal to a price currently obtainable in the spot market for the underlying asset; F_t is a known variable equal to the current futures or forward contract price quoted on a futures exchange or over-the-counter market; but that $E(S_t)$ is an unknown variable which converts into S_0 at some future point in time. Symmetric relationships shall be designated by the symbol $\leftrightarrow$ whereas asymmetric relationships shall be designated by the symbol $\rightarrow$ or $\leftarrow$ with the arrow designating which direction risk premia is transferred [e.g., (H_S) $\rightarrow$ (A_Δ), means that net risk premia was transferred from short hedger to speculators, resulting in net excess returns for speculators]. Additionally, rising markets or prices are designated by the symbol ($\uparrow$) whereas declining markets or prices are designated by the symbol ($\downarrow$). Last, counter-trend strategies are designated by C, and trend-following strategies are designated by T; for example, $(H_S)\cdot C$ means a short hedger following a counter-trend strategy.

Sprugin calls the inclination for long and/or short hedgers to enter into a hedge transaction based on increases or decreases in the price of the underlying commodity, the 'hedging response function.' This function results in either a symmetric outcome where the hedging demand from both short and long hedgers is in equilibrium, or four possible asymmetric scenarios stemming from either of two predispositions: (i) an inclination to either lock in profits or let profits run, which behaviorally reflects a *greed* factor in varying degree; or (ii) an inclination to protect against losses or let losses run, which behaviorally reflects a *fear* factor in varying degree.[23] This framework offers insight into whether trend-following strategies or counter-trend strategies (Spurgin calls the latter 'responsive strategies') will be successful over time. The key to Spurgin's model is the hedging response function as determined by the four asymmetric scenarios described below:

In scenario [A1], a rise (↑) in commodity price (which is beneficial to producers) generates more initiative from producers (H_S) to lock in higher prices, resulting in a net short hedging position. Theoretically, consumers (H_L) who are harmed from this scenario have deferred hedging, either hoping that prices decrease or believing that the price increase can be passed along to customers.

In scenario [A2], a drop (↓) in commodity price (which is beneficial to consumers) generates more initiative from consumers (H_L) to lock in lower costs, resulting in a net long hedging position. Theoretically, producers (H_S) who are harmed from this scenario have deferred hedging, either hoping that prices will increase or are willing to absorb reduced margins to the customers' benefit.

If the hedging response for a market follows scenarios [A1] or [A2], the net hedging position will follow a counter-trend strategy. This implies that: (i) for scenario [A1] higher prices (↑) will result in producers (H_S) moving more quickly to lock in margins than consumers (H_L) move to stem margin pressures, resulting in a net short hedge position; or (ii) for scenario [A2] lower prices (↓) will result in consumers (H_L) moving more quickly to lock in margins than producers (H_S) move to stem margin pressures, resulting in a net long hedge position. Since the net speculative position (A_Δ) is simply the inverse of the hedge positions, we will observe a trend-following strategy in the net speculative position. Hence, in reaction to these kinds of hedger responses, higher prices (↑) will theoretically result in a net-long trend-following speculative position for scenario [A1], and lower prices (↓) will theoretically result in a net-short trend-following speculative position for scenario [A2]. These scenarios can be described by the following equations: scenario [A1] is equivalent to $(\uparrow) = (H_S) \rightarrow (A_\Delta)$, resulting in $(H_S)\cdot C \rightarrow (A_\Delta)\cdot T$; and scenario [A2] is equivalent to $(\downarrow) = (H_L) \rightarrow (A_\Delta)$, resulting in $(H_L)\cdot C \rightarrow (A_\Delta)\cdot T$. In each scenario, risk premia or excess returns is transferred to (A_Δ).

[23] 'Greed' and 'fear' are behavioral factors which we suggest, but which were not presented in Spurgin's original article. Rather, Spurgin frames the response functions in economic rather than behavioral terms.

In scenario [B1], the opposite of [A1] occurs, and a rise (↑) in commodity price causes consumers (H_L) to be more concerned about guarding against margin pressure than producers (H_S) are about locking in profits, hence a net long hedging position is established. Producers (H_S) benefit from this scenario, and therefore are less inclined to hedge because there is no need to protect against a loss, and/or because they want exposure to the potential of higher prices and increased profitability.

In scenario [B2], the opposite of [A2] occurs, and a drop (↓) in commodity price causes producers (H_S) to be more concerned about guarding against margin pressure than consumers (H_L) are about locking in lower costs, hence a net short hedging position is established. Consumers (H_L) benefit from this scenario, and therefore are less inclined to hedge because there is no need to protect against a loss, and/or because they want exposure to the potential of lower prices and better margins.

If the hedging response for a market follows scenarios [B1] or [B2], the net hedging position will follow a trend-following strategy. This implies that: (i) for scenario [B1] higher prices (↑) will result in consumers (H_L) moving more quickly to stem losses than producers (H_S) move to lock in increased margins, resulting in a net long hedge position; or (ii) for scenario [B2] lower prices (↓) will result in producers (H_S) moving more quickly to stem losses than consumers (H_L) move to lock in increased margins, resulting in a net short hedge position. Again, since the net speculative position (A_Δ) is simply the inverse of the hedge positions, we will observe a counter-trend strategy in the net speculative position. Hence, in reaction to these kinds of hedger responses, higher prices (↑) will theoretically result in a net-short counter-trend speculative position for scenario [B1], and lower prices (↓) will theoretically result in a net-long counter-trend speculative position for scenario [B2]. These scenarios can be described by the following equations: scenario [B1] is equivalent to (↑) = (H_L) → (A_Δ), resulting in $(H_L)\cdot T \rightarrow (A_\Delta)\cdot C$; and scenario [B2] is equivalent to (↓) = (H_S) → (A_Δ), resulting in $(H_S)\cdot T \rightarrow (A_\Delta)\cdot C$. In each scenario, risk premia or excess returns is transferred to (A_Δ).

As can be inferred by the above four asymmetric hedging response functions, it is possible, if not typically the case, that: (i) reaction to rising prices can simultaneously generate either [A1] or [B1] response functions from individual agents; and likewise, (ii) reaction to falling prices can simultaneously generate either [A2] or [B2] response functions from individual agents. While the model assumes that the "net hedging position" results in one scenario/function prevailing over another, in reality the responses of individual agents exert influence over price movement in line with the Sonnenschein-Mantel-Debreu theorem. Therefore, it is difficult, if not impossible, to know at any particular time what the true aggregate behavior or "net hedging response" is within a bullish or bearish market cycle. Such knowledge of market behavior can only be ascertained on a case-by-case basis for any particular market participant.

Spurgin notes that certain conditions are necessary for aggregate speculator positions to be profitable. However, such conditions are not sufficient to prove the existence of positive returns. Accordingly, if these conditions are not present in a given market, then it will be difficult to argue that the aggregate speculator position has a positive expected return and earns risk premia. Spurgin notes, as we also previously note, that returns may be asymmetric in the short term, and "some may gain and some may lose. All may even gain for a short period of time. But absent these conditions the most likely outcome is a zero sum game for speculators." In other words, absent the following conditions, the aggregate outcome over the long run is symmetric. Below are Spurgin's conditions:

(1) *Both consumer and producers of a product have a strong economic rationale to hedge*. If there is no commercial hedging in a market, and all risk is held by speculators, the futures market will be in equilibrium and the net return to speculators is assumed to be zero before transaction costs. On the other hand, if one side of the market has an economic rationale to hedge, but the other side does not, then the futures market is a pure insurance market. Spurgin describes this situation as analogous to life insurance, "where there is a natural economic rationale to buy such insurance, but nobody with a natural reason to take the other side of the trade. [Since] insurance is priced based on expected loss, [it] must be priced above fair value in order to compensate the insurer for bearing the risk and the costs [of administrating] the policies."[24] From this line of reasoning, one can logically assume one or the other of the following scenarios/market conditions will be prevalent:

[A1/B2] A futures market populated mainly with producers (H_S) but few or no consumers (H_L) will result in speculators (A_Δ) having a net long futures position. As a consequence, backwardation will be the prevailing market condition, where the futures contract is consistently priced below the expected future spot price. This relates to the logical statement: if $(H_S) > (H_L)$, then $F_t < E(S_t)$.

[A2/B1] A futures market populated mainly with consumers (H_L) but few or no producers (H_S) will result in speculators (A_Δ) having a net short futures position. As a consequence, contango will be the prevailing market condition, where the futures contract is consistently priced above the expected future spot price. This relates to the logical statement: if $(H_S) < (H_L)$, then $F_t > E(S_t)$.

Successful futures markets attract both producers and consumers, with one side at any point in time, having a net greater influence on market price. At the same time, it is assumed that symmetrically opposed hedgers offset their positions, such that $(H_S) \leftrightarrow (H_L)$. However, the hedging response function also implies that in a rising or falling markets, producers (H_S) and consumers (H_L)

[24]Spurgin suggests that "The first two conditions rule out the possibility that the demand for risk management in the asset is not a pure insurance market. Futures contracts can serve this function, but there are other risk management vehicles that are better suited to this task. In order for futures markets to provide a suitable vehicle for risk management, the net hedging position cannot always be on the same side of the market."

respond with different behavioral tendencies to the same directional bias—either scenarios [A1] and [B1], or scenarios [A2] and [B2]. Depending on which group of hedgers has the net greater influence, the market will accordingly be in backwardation or contango. The following examples illustrate this:

For example, if (↑), then either [A1] $(H_S)\cdot C \rightarrow (A_\Delta)\cdot T$ or [B1] $(H_L)\cdot T \rightarrow (A_\Delta)\cdot C$; therefore, if $(H_S) > (H_L)$, then $F_t < E(S_t)$ and $(H_S)\cdot C \rightarrow (A_\Delta)\cdot T$; otherwise, if $(H_S) < (H_L)$, then $F_t > E(S_t)$ and $(H_L)\cdot T \rightarrow (A_\Delta)\cdot C$; moreover, for any situation where $(H_S) = (H_L)$, then $F_t = E(S_t)$ and $(H_S) \leftrightarrow (H_L)$.

Alternatively, if (↓), then either [A2] $(H_L)\cdot C \rightarrow (A_\Delta)\cdot T$ or [B2] $(H_S)\cdot T \rightarrow (A_\Delta)\cdot C$; therefore, if $(H_L) > (H_S)$, then $F_t > E(S_t)$ and $(H_L)\cdot C \rightarrow (A_\Delta)\cdot T$; otherwise, if $(H_L) < (H_S)$, then $F_t < E(S_t)$ and $(H_S)\cdot T \rightarrow (A_\Delta)\cdot C$; moreover, for any situation where $(H_L) = (H_S)$, then $F_t = E(S_t)$ and $(H_L) \leftrightarrow (H_S)$.

(2) *It should be difficult to pass along changes in the price of a commodity to users of that product*. In an elastic market, where the price of a 'consumable/transformable asset' can be easily passed along to consumers, a business will not be inclined to long hedge (i.e., lock in prices for input commodities or factors) or short hedge (lock in prices of the business' output commodities or products). Only in an inelastic markets, where changes in the price of input commodities/factors or output commodities/products are difficult to pass along in the form of higher prices to customers, will a business have an economic rationale to hedge. This second condition taken together with the first condition, describe an environment where a speculator can reasonably expect to be compensated by hedgers for providing liquidity and bearing risk. To help clarify how this second condition operates, we present two separate examples of inter-related scenarios below. These situations also provide an illustration of how social reflexivity influences the model's outcome.

> Scenario Series 1
>
> Let's assume that a baker, technically a middleman that functions as both consumer and producer, has little incentive to hedge (H_L) against an increase in wheat prices because it can pass on higher bread prices to its customers (i.e., elastic market).
>
> Alternatively, a farmer decides that hedging (H_S) is his best option in order to lock in rising wheat prices and the resulting increased margins. This will allow him to meet the rising fixed costs of running his farming operations (i.e., inelastic market).
>
> Combined scenarios reflect: $(H_S) > (H_L)$ and $F_t < E(S_t)$, resulting in $(H_S)\cdot C \rightarrow (A_\Delta)\cdot T$
>
> Continuing with the example, the baker borrows money at floating rates which later increase. For the baker, this situation is inelastic since he/she would have difficulty passing along the higher interest expense in the form of higher bread prices. This is because competitors that borrowed at fixed rates will not have that motivation to raise bread prices. For this reason, the baker may be inclined to hedge (H_L).

Assuming rates (↑) and $(H_L) > (H_S)$, then $F_t > E(S_t)$ resulting in $(H_L)\cdot T \rightarrow (A_\Delta)\cdot C$; or
Assuming rates (↓) and $(H_L) > (H_S)$, then $F_t > E(S_t)$ resulting in $(H_L)\cdot C \rightarrow (A_\Delta)\cdot T$.[25]

Since the original scenario describes rising interest rates after the baker borrowed money at floating rates, market conditions from baker's perspective is $F_t > E(S_t)$, since baker is "chasing" a rising interest rate market. However, from the aggregate perspective of all agents in the market, conditions may in fact be $F_t < E(S_t)$.

Scenario Series 2

Let us assume that a utility which produces electricity (i.e., producer) is regulated so that it cannot easily pass along its production price to consumers. This describes an inelastic market Consumers have no need to hedge electricity prices because the government has capped the amount that can be charged to them. From the consumer's perspective, the situation deters hedging (H_L) on the consumption side.

Hence, exactly because this regulated environment caps how much the utility can charge its customers, the situation also deters hedging (H_S) on the production side.

Combined scenarios reflect: $(H_S) = (H_L)$, then $F_t = E(S_t)$ and $(H_S) \leftrightarrow (H_L)$

On the other hand, if this same utility must purchase coal at market price in order to generate electricity, then it has a strong incentive to lock in rising coal prices (i.e., an input factor) by hedging (H_L). Coal production is an inelastic market from the utilities' perspective, but an elastic market from the coal producer's perspective.

If the coal producer (H_S) is less inclined to hedge than the utility, a rising market will likely reflect: $(H_S) < (H_L)$, then $F_t > E(S_t)$ and $(H_L)\cdot T \rightarrow (A_\Delta)\cdot C$; and a declining market will also reflect: $(H_S) < (H_L)$, resulting in $F_t > E(S_t)$ and $(H_L)\cdot C \rightarrow (A_\Delta)\cdot T$

If the coal producer (H_S) is more inclined to hedge than the utility, a rising market will likely reflect: $(H_S) > (H_L)$, then $F_t < E(S_t)$ and $(H_S)\cdot C \rightarrow (A_\Delta)\cdot T$; and a declining market will also reflect: $(H_S) > (H_L)$, resulting in $F_t < E(S_t)$ and $(H_S)\cdot T \rightarrow (A_\Delta)\cdot C$

(3) *Demand for long and short hedging should respond differently to changes in the price of the good*. As examined thoroughly in the first part of this section on the hedging response model, the 'hedging response function' describes how the "quantity of hedging" demanded by long and short hedgers changes as the price of the underlying good changes. This hedging response function "eliminates" the possibility that the net hedging position will always be in balance. This is supported by the first condition which suggests that: "If long and short hedgers have the same response function, then the net hedge position will always be zero. [As a result,] the net speculative position will also be zero and the returns to speculative capital in that market will be a zero sum game. [Hence,] only if the net hedging position is sometimes long, sometimes short and sometimes in balance will a futures market provide a suitable vehicle for hedgers to manage risk, and for speculative capital to earn [risk premia with a] positive expected return."

[25] Both of the hedging response functions are in reaction to bullish or bearish contango market conditions. Note that the 'congenital weakness' attribute is not relevant to these kind of net hedging scenarios, and that Spurgin's model does not place any additional weighting to Keynes' normal backwardation constraint.

(4) *A well-developed arbitrage relationship between the underlying asset and the futures market.* This condition ensures that the net hedging position in a particular asset will be efficiently transmitted to its exchanged-traded futures contract, so speculators do not need to trade in off-exchange risk management vehicles in order to purchase the excess risk. The objective of the arbitrageur is to survey the various instruments available on that particular asset, such as forwards, swaps and options, or even the underlying cash asset itself, and take relative value positions linking the futures markets to these other hedging vehicles. Thus, Spurgin casts the arbitrageur in a somewhat expanded role relative to the one described by the classic arbitrage or simplified arbitrage models. As applied to Spurgin's model, the existence of arbitrageurs allows the market to assume that net hedging positions, the difference between the net long positions of long hedgers and the net short positions of short hedgers, will be transferred to the futures contract. As long as net hedging risk is transferred to the futures market, a speculator can take positions in futures even if the typical hedger is primarily using other vehicles. However, if the delivery specifications of the contract are too narrowly defined or if liquidity in the contract is too thin, arbitrage will be difficult, and the risk embedded in OTC hedging vehicles or the actual cash commodity will not be efficiently transferred to the futures market.

Interestingly, the last point of this condition supports an uncommon hypothesis that the less liquid and more distant future delivery contracts require hedgers to offer additional risk premia for such contracts in order to sufficiently attract speculators. Consequently, an argument can be made that less liquid contracts potentially offer more risk premia and therefore are more lucrative to speculators, as suggested by the hedging response model. In practice, however, speculators tend to actively trade the more liquid contracts (e.g., NYMEX WTI crude oil rather than MCX India WTI crude oil; note different delivery points) and focus on the nearby front month contract.

To reiterate, Spurgin's conditions are not sufficient to prove the existence of positive returns, but if these conditions are not present, then it will be difficult to argue excess risk premia. What we find interesting about Spurgin's hedging response model, is that, unlike the classic arbitrage and simplified arbitrage models, which are conventionally considered within the context of an equilibrium state, this model is premised on the condition of an asymmetric net hedging position in order to create the opportunity for speculators to capture excess risk premia and positive expected returns. It is essentially a reflexive model, whose conceptual origin is a state of disequilibrium. While Spurgin does not present the hedging response functions as such, his set of mechanisms in our opinion represents a behavioral risk management model. Further, as already theorized in regards to the other models, the hedging response model does not operate independently in a vacuum. Rather, this and all models presented previously are inter-related, with each reflecting certain aspects and dynamics within the total futures market paradigm. What ties them together is the theory of reflexivity and feedback loops within and between each other.

IV. Conclusion: The Beta of Futures is Behavioral

"Keynes believed that fundamental uncertainty is a crucial element in any economic processes. And that under most circumstances, even if probabilities could be estimated, they [are] meaningless for long period decision making. The nature and power of market forces cannot deal with the unpredictability of the long run, and relying on them to do so will lead to incomplete information."[26] Ric Holt's *Post-Keynesian* articulation of Keynes's ideas on uncertainty and predictability underscores our thesis. From our perspective as practitioners, it seems obvious that the truth is somewhere in the middle rather than at the ideological extremes of neoclassical ergodic systems. Models are not exclusive and each reveal underlying *qualities* within the aggregate wealth portfolio of all agents in the *global* economy. However, unto themselves, they do not provide that one universal asset pricing solution which encompasses all cross-sectional variations. *That model* is akin to seeking the holy-grail. If you are seeking the true name of god, then watch the movie π. What these models convey is an insightful understanding, provided one accepts that in the *real world* agents are irrational, that markets drift from disequilibrium to equilibrium and back, and inputs/outputs are reflexive. With that in mind…

Our investigation shows that the research is inconclusive with respect to modeling the sources of returns in the commodity futures markets, largely because these models have inherent shortcomings in being able to pinpoint a definitive source of structural risk premium within the complexity of such markets. We hypothesize that the classic arbitrage pricing theory contains circular logic, and as a consequence, is subject to reflexivity. If such model is in fact a reflexive model, we propose that its *natural* state is disequilibrium, not equilibrium. We extended this hypothesis to suggest that the term structure of the futures price curve, while indicative of a potential roll return benefit (or detriment), in fact implies a complex and reflexive series of roll yield permutations which amplifies into an exponential number of potential market scenarios. Similarly, Spurgin's 'hedging response functions' models behavioral risk management mechanisms, and therefore, corroborates social reflexivity. These models do not operate to the exclusion of the other, nor exclusively from each other; instead, such models are inter-related and each reflect certain aspects and dynamics within the total futures market paradigm. In fact, the various commodity asset pricing models interact through reflective feedback between each other, as well as internal feedback within themselves. Hence, we posit that the combination of models we investigated support a *post-Keynesian* view that the world is messy and uncertain.

[26] Ric Holt, "What is Post Keynesian Economics?" http://cc.shu.edu.tw/~tsungwu/holt.htm

Further, we surmise that these models, which are conventionally regarded as validation for persistent and replicable sources of return in the commodity futures markets, may be widely misunderstood. There is no doubt that the futures markets offer vicarious economic benefits, such as price discovery, price liquidity, reduced price volatility and therefore increased capacity utilization. But again, such attributes benefit the businesses that utilize the futures markets, as well as the economy as a whole, not necessarily those economic agents called speculators. If there is a *beta* that speculators capture from the commodity futures markets, it is likely sourced from irrational behavior and market disequilibrium. And if that is the case, then what does that say about those commodity asset pricing models founded on the premise of perfect markets and rational expectations? Alternatively, Spurgin's hedging response model supports the idea that, if the *wisdom of crowds* is balanced, then trends will evolve which the speculator can take advantage of (in a leveraged manner, we will add). However, if the *madness of crowds* goes too far one way or the other, then the speculator will step in with a counter-trend strategy. If correct, such speculator will be rewarded with sufficient positive returns to have made the bet worth the risk—but the result may be asymmetric! Others will have lost, and on the whole the summative profit-loss outcome will remain essentially symmetric—theoretically a zero sum game.

We began this article by discussing *alpha*, which in a general sense is intended to measure a manager's skill-based returns (and which in futures trading should supposedly be related to asymmetric returns within a symmetric paradigm). If one understands the simplistic equation for calculating alpha, then the fundamental question, and a much more interesting one, is "what is the appropriate beta index?" The beta is required to determine alpha, and the active investment strategy under question must technically respond to the same return drivers as the benchmark it is compared to. This leaves open the question as to whether institutions, through sophisticated financial engineering, can truly capture in a passive way all possible sources of return in the global economy. *Or*, does some aspect which the industry loosely calls alpha (i.e., skill-based returns) always remain outside the grasp of such institutions' "arbitrary" models of beta?

We note there are institutional pressures and economic incentives which lead to the usage of benchmarks and passive indices, and modeling provides justification for creating and bringing to market many innovative but untested 'beta replication' products. Exchange traded funds (ETFs) and exchange traded notes (ETNs) based on traditional investments for the most part are justifiable since their underlying investments are 'capital assets.' In addition, equity and fixed income proxies can serve a valuable purpose in measuring traditional portfolio risk and return on a relative (but imperfect) basis. More importantly, why should an investor pay exorbitant fees for so-called alpha when that investor can obtain the same or similar asset exposure through an inexpensive beta product? Financial innovation should always be encouraged, and in response financial institutions will continue with their efforts to securitize all identifiable combinations of assets and replicable trading strategies into "smart" exotic beta products, commodities included. But do these marketable investment vehicles, derived from Guassian distributions based on regressions of *hypothetical* performance data, always serve investors' best interest with respect to constructing diversified long-only passive portfolios? We contend that index vehicles based on commodity assets will prove over the long run (beyond the current secular bull market in commodities) to *not* be the reliable and consistent source of excess premia or positive expected returns as is proposed, much less a means to gauge the relative performance of managed futures.

At the same time, in the larger scheme of things, we recognize that the product development process is also a reflexive economic activity. With respect to managed futures benchmarks and commodity-based indices (e.g., long-only, long-short, passive or "smart") vying for the *beta crown*, we declare "to the victor goes the spoils." However, investors should be careful for what they wish for, as it is likely that over the long-term returns in this space will revert to the mean, which is zero or near zero. At minimum, the legacy of academic research is inconsistent and has not yet proven conclusively that a persistent structural risk premium exists within the commodity futures market. This lingering concern is historically borne out by the waning returns of various commodity trading advisor benchmarks, such as the Barclay CTA Index or the S&P Managed Futures Index, especially if one eliminated survivorship bias from these composite "hypothetical presentations." In light of such observations, we anticipate that a similar fate will eventually result from the plethora of commodity-based ETFs and ETNs coming to market, and long-term systematic risk will prove to be zero for these investment products too.

Correspondingly, we deduce that the concept of a *futures market beta*, specifically futures based on commodity interests, is at worst an illusion, and at best behavioral. It may be an illusion because as Greer (1997) noted, commodities are not capital assets but instead "consumable/ transformable assets," and as such, the *beta of commodities* is transferred and captured by the businesses that produce, consume and transform them. Hence, over the long-term speculative returns in the commodity markets are likely to revert to the mean, which is near zero, if not less than zero due to commissions. But that also doesn't mean there cannot be a secular bull market in the spot returns of underlying commodities which can be captured via futures trading. Nevertheless, managed futures, in our opinion, is an observable materialization of behavioral finance, where risk, return, leverage and skill operate un-tethered from the anchor of an accurate representation of *beta*. In other words, it defies rational expectations equilibrium, the efficient market hypothesis and allied models—the CAPM, arbitrage pricing theory or otherwise—to isolate a persistent source of return *without that source eventually slipping away*. So assuming that futures speculation is a zero sum game, then are returns from managed futures other than zero—alpha?" The answer to that is "no" if an acceptable proxy benchmark for managed futures doesn't exist. Therefore, it is inappropriate for a futures speculator to make a claim of positive *alpha* simply because investment returns are greater than the risk free rate, unless the portfolio is risk-free. Speculating in commodity futures is *not* risk-free. But that doesn't mean that certain traders do not have that ability the "old school" once called an *edge*—the adept consistently capture risk premia from the wisdom of crowds and/or the madness of crowds.

V. Afterword: Applicability to Managed Futures

We titled this research article "Is Managed Futures an Asset Class; The Search for the Beta of the Commodity Futures" with a particular purpose in mind. The following essay links that intention.

In accordance with the principles of modern portfolio theory, which states that, the addition to an existing portfolio of an uncorrelated asset with a positive expected return will improve that portfolio's risk-adjusted return, sophisticated investors have increasingly sought to diversify their portfolio through the use of alternative investments. An 'alternative investment' is generally regarded as supplementary assets or trading strategies other than long-only exposure to 'traditional assets' such as stocks, bonds and/or cash. Alternative investments include various assets such as commodities, currencies, emerging markets, private equity and real estate, etc., as well as various trading methodologies such as convertible arbitrage, distress securities, global macro, long-short equities, managed futures, premium capture, risk arbitrage, short selling, etc. A commonly stated assumption by adherents of alternative investments is that it has a low to negative correlation compared to traditional investments, historical performance which reflects the potential for attractive positive expected returns, and is capable of acting as a hedge against inflation. The combination of these factors suggests that, within the diversification tenets of modern portfolio theory, a strong case can be made for the inclusion of alternative investments in traditional portfolios consisting mainly of equities and fixed income.

Whereas Harry Markowitz's seminal 1959 work, "Portfolio Selection: Efficient Diversification of Investments," established modern portfolio theory (MPT) as a mainstay investing precept based on an analysis of traditional investments, subsequent academic studies which investigated the inclusion of alternative investments along the 'efficient-frontier'[27] at first centered around

[27] A key premise of Markowitz's MPT is the "efficient frontier," which is a collection of every possible asset combination (i.e., portfolios) offering the highest expected return for a given amount of risk.

commodity investments. A key impetus for such studies to initially focus on commodities was the availability and length of historical data.[28] Distinctively, these markets have been formalized since the 1800s, and are also regulated by the U.S. Federal Government; initially under the 1921 Capper-Tincher Act (Grain Futures Act), and later under the Commodity Exchange Act of 1936, which eventually led to the formation of the Commodity Futures Trading Commission (CFTC) in 1974. It is under the auspices of the CFTC that managed futures developed into its own niche specialty.

One of the first academic studies showing the desirable features of commodity futures in portfolio diversification was co-authored by Zvi Bodie, Associate Professor, Boston University, and Victor Rosansky, doctoral student. Published in 1980 by *Financial Analysts Journal*, "Risk and Return in Commodity Futures" concluded that commodity futures were "naturally" negatively correlated with stocks and bonds, and were a good inflation hedge. The analysis was based on a 40% allocation to a buy-and-hold "benchmark" portfolio of commodity futures, consisting of all major commodities traded in the U.S. futures markets from December 1949 to December 1976, combined with a portfolio of 60% common stocks as represented by the S&P 500 index. In comparison to an all stock portfolio, this hypothetical commodity portfolio produced a reduction in return variability by one-third, without sacrificing return. Further, the long-term nature of the study also revealed that downside risk may be less for a commodity portfolio than for stocks or bonds.[29]

In Spring 1983, Bodie followed this joint paper with a study published in *The Journal of Portfolio Management*, entitled "Commodity Futures as a Hedge against Inflation." This study, which assumed a strict buy-and-hold strategy for a commodity futures portfolio based on annual returns from 1953 to 1981, pointed out that under such a strategy the only hope for long-term, positive return is through unanticipated increases in spot prices of the underlying commodities. In doing his portfolio analysis, Bodie made the conservative assumption that long-term, real rates of return from the portfolio were between 0% and 2% (although his study exhibited real returns of 5.69% for the 29 years analyzed). The result was that at any level of risk, a higher mean rate of return was achieved, and a favorable shift to the efficient frontier occurred from the use of commodity futures. Further, the portfolio tended to do well "precisely" in those years when

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[28] The history of modern futures trading began with the establishment of the Chicago Board of Trade (CBOT) in 1848 by 82 merchants. Futures contracts were developed by the CBOT in 1865, and the CBOT also introduced the first open-outcry trading pit in 1870. The predominantly traded commodities were wheat and corn, but traders figured that if dealing on a forward basis worked well for these two goods, they could establish contracts on other products as well. Subsequently, additional exchanges were established including the Chicago Butter and Egg Board in 1898, which evolved into the Chicago Mercantile Exchange (CME) in 1919. In 1972 the CME launched the first financial futures contracts offered on seven foreign currencies.

[29] Carl C. Peters, Editor. "Managed Futures, Performance, Evaluation and Analysis of Commodity Funds, Pools and Accounts," Chapter Two. Probus Publishing Company, 1992.

inflation was high, with the exception of 1981, a year of "unanticipated deceleration in the rate of inflation" as "compared to the forecasts of most experts at the beginning of the year."[30]

While the aforementioned vanguard studies by Bodie and Rosansky (1980), and Bodie (1983) focused on passive commodity futures exposure, it was John Lintner, Professor, Harvard University, who first analyzed the effect on a traditional stock and/or bond portfolio of actual performance in an actively managed alternative investment strategy, specifically a subset of alternative investments known as managed futures. His May 1983 paper presented to the *Annual Conference of the Financial Analysts Federation*, entitled "The Potential Role of Managed Commodity-Financial Futures Accounts (and/or Funds) in Portfolios of Stocks and Bonds," is considered to be a landmark study.

Linter's study was based on monthly returns for 42 periods from July 1979 through December 1982, encompassing the composite performance of fifteen "futures-accounts managers,"[31] eight "publicly offered commodity funds,"[32] averages of all stocks listed on the NYSE and AMEX, the Salomon Brothers high grade corporate bond index, U.S. Treasury Bills, and the Consumer Price Index. Results indicated that the ratio of return to risk (standard deviation) was higher for a substantial percentage of trading advisors and commodity funds than it was for stock, bond and "traditional stock and bond portfolios." Lintner concluded that portfolios of stocks and/or bonds diversified with managed futures (i.e., trading advisors and commodity funds) showed materially less risk at every possible level of expected return than portfolios of stocks and/or bonds alone. These results are true for both actual returns and returns adjusted for inflation. Quoting Lintner, "Indeed, the improvement from holding efficiently selected portfolios of managed futures accounts or funds is so large, and the correlations between the returns on futures portfolios and those on the stock and bond portfolios so surprisingly low (sometimes even negative), that the return/risk tradeoffs provided by augmented portfolios… clearly demonstrate tradeoffs available from portfolios of stocks alone (or portfolios of stocks and bonds). Moreover, they do so by very considerable margins."[33]

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[30] Carl C. Peters, Editor. "Managed Futures, Performance, Evaluation and Analysis of Commodity Funds, Pools and Accounts," Chapter Three. Probus Publishing Company, 1992.

[31] Lintner's paper alternatively describes these investments as "managed accounts of trading advisors in the commodity and financial futures markets," and cites that monthly return data is based on composite account performance reports filed with the CFTC and SEC. While no reference is made to Commodity Trading Advisors (CTAs), it is assumed that the returns are derived from CTA performance reporting.

[32] Lintner's paper alternatively describes these investments as "publicly traded futures funds," and cites that monthly return data is based on compiled public reports by Jay Klopfenstein of Norwood Securities in Chicago and Frank S. Pusatri in New York as reported in Managed Account Reports, Columbia, MD. While no reference is made to Commodity Pool Operators (CPOs), it is assumed that the returns are derived from CPO performance reporting at the time.

[33] Carl C. Peters, Editor. "Managed Futures, Performance, Evaluation and Analysis of Commodity Funds, Pools and Accounts," Chapter Four. Probus Publishing Company, 1992.

So how did managed futures evolve? Actively managed alternative investment strategies were first pioneered by Richard Donchian in 1948 with the establishment of the first commodity fund, and with Alfred Winslow Jones who started the first hedge fund in 1949. The success of these funds spawned what is now commonly referred to as the hedge fund industry and the managed futures industry. According to an April 1999 report by the *President's Working Group on Financial Markets* on "Hedge Funds, Leverage, and the Lessons of Long-Term Capital Management," the term 'hedge fund' is not statutorily defined, and encompasses any pooled investment vehicle that is privately organized, administered by professional investment managers, and not widely available to the public. 'Managed futures' refers to professionally managed assets in commodity and financial futures including options on futures, and to a lesser extent, forward contracts. Management of client assets is directed by Commodity Trading Advisors (CTAs) and Commodity Pool Operators (CPOs), and facilitated by Futures Commission Merchants (FCMs) and Introducing Brokers (IBs). These entities are required to be registered with and regulated by the Commodity Futures Trading Commission (CFTC) under the legal framework of the Commodity Exchange Act of 1936.

Circa 2006, the hedge fund industry is estimated to have between US$1.2 trillion[34] and US$2.4 trillion[35] in investments, as compared to the managed futures industry which is estimated to have approximately US$170 billion[36] assets under management. Despite the size and status of hedge funds relative to managed futures, the latter's impact upon the alternative investment space is writ large in two significant and related ways: first, managed futures unlike its brethren hedge funds operate in a highly regulated environment; second, this same regulated environment which imposes disclosure and reporting requirements, compelled the data on managed futures to be made public, which in turn helped academics advance early studies on alternative investments, prior to developing any substantial research on hedge funds. In effect managed futures helped institutionalize the space.

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[34] Assets under management of the hedge fund industry totaled $1.225 trillion at the end of the second quarter of 2006 according to data by Hedge Fund Research, Inc. This was up 20% on the previous year and nearly twice the total three years earlier. In a separate study published by Institutional Investor News and HedgeFund.net, total estimated assets for the industry grew by 24% in 2006 to a total of $1.9 trillion. Performance is said to have accounted for 33% of the total increase.

[35] SEC Communication (Jun. 23 2006). Adviser population snapshot (from Forms ADV): As of [Jun. 23 2006], there are 10,627 investment advisers registered with the SEC. In total, these advisers manage $32 trillion in investor funds. Of the advisers registered with the SEC, 2,533 indicate that they manage a hedge fund, or approximately 24% of the total number of advisers registered with the SEC. The hedge fund advisers registered with the SEC advise a total of 13,876 hedge funds, and the total assets under management in these hedge funds is $2.4 trillion (or approximately 8% of total assets for all registered advisers). Since January 1, 2005, some 1,260 hedge fund advisers became newly-registered with the SEC.

[36] In a press release dated February 14, 2007, assets under management in the managed futures sector grew from US$130.6 billion to US$170 billion during 2006, a 30% increase, according to data collected by The Barclay Group, a company founded in 1985 which actively tracks managed futures programs.

It is noted that the present day situation for hedge funds is much improved and allows for the collection of significant data points, making it now possible to perform meaningful analysis of this universe. Historically, however, this wasn't always the case as hedge funds in prior times were, and many are still to this day, highly secretive. It is therefore ironic that the smaller (in terms of assets managed) segment of alternative investments, managed futures, played such an early and important role in the development of academic research which eventually led to the institutionalization of the broader alternative investment space. All the same, as pointed out by Schneeweis and Spurgin (1996), the continuing relatively low level of investment in managed futures is due to the fact that institutional investors require both a theoretical basis for their investment in nontraditional investments, as well as supporting empirical results. For stocks and bonds, both single and multi-factor theoretical models, as well as empirical tests of return formation exist. Evidently, however, return expectation models and empirical investigations of the futures market based on derivations of the 'normal backwardation' theory of Keynes (1930) have produced inconsistent results.[37]

Recently, though, there is a proliferation of new studies on the futures market since the new millennium, many which are supportive, if not presumptive, of the idea of a 'structural risk premium' in the commodity futures market. Is there a reflexive correlation between the past few years' commodity bull market and such suppositions? At the risk of sounding blasphemes, we repeat here what one respected academic had to say about the subject: "I have to admit that I have no faith at all in financial models or in their empirical tests. The industrial-scale production of both over decades with no generally agreed-upon conclusions is probably the best evidence that the whole enterprise is more geared towards furthering academic careers than it is to finding anything that might resemble truth. I'd be more inclined to argue that the markets are social and cultural phenomena that are so driven by complex reflexive loops that mathematical and statistical techniques are wholly unable to sort them out." For better or worse, our research seems to support this point of view.

So how does managed futures relate to our research on the classic and simplified arbitrage models, theory of roll yield permutations, and Spurgin's (2000) hedging response model? Simply, managed futures is where the theoretical becomes *real world*. Those agents called 'speculators' in academic models are best represented in the real world by commodity trading advisors (CTAs). And while real-life speculators who do not hold themselves out to the public as such certainly exist, CTAs (and to a lesser extent commodity pool advisors) provide our best window into the *actual*

[37] In the context of Keynes (1930) theories, Houthakker (1957) finds empirical support for normal backwardation in corn, cotton and wheat futures, while Rockwell (1967) contradicts this study using a broader data set. Telser (1958) also disagrees with the findings of Houthakker, while Gray (1961) finds results consistent with Telser.

trading practices and performance data of *actual* speculators. That said, experienced practitioners in this investment specialty are all too aware of the idiosyncratic nature of this segment of alternative investments. First, there is a myriad of sub-specialties within managed futures. For example, there are CTAs who are considered trend-followers, while others trade counter-trend; some focus on calendar spreads, others trade options only; some make decisions based on fundamentals, while others use technical analysis; likewise some trade discretionarily, others systematically; and *not* finally, some trade a diversified basket of futures, while others just focus on financials, or just commodities, or specialize in a commodity sector such as metals or energies or grains, etc. And then there is every kind of mixture of the above trading styles, time horizon and portfolio construction. Add to this confusion the question of whether or not there is a structural risk premium, a persistent source of return, or *pure alpha*, and you have "a riddle, wrapped in a mystery inside an enigma."

Answering our own question *is managed futures is an asset class*? It is anything, but… If anything, it is the "anti-asset class." It is an observable materialization of behavioral finance, where risk, return, leverage and skill operate un-tethered from the anchor of an accurate representation of *beta*. In other words, it defies rational expectations equilibrium, the efficient market hypothesis and allied models—the CAPM, arbitrage pricing theory or otherwise—to single-handedly isolate a persistent source of return *without that source eventually slipping away*.

Still, the financial industry seems driven to sail forward and seek answers in platonic math. In comparing modern finance with behavioral finance, Frankfurter and McGoun (2002), in their article "Resistance is Futile: The Assimilation of Behavioral Finance," make the following astute observation which can be similarly applied to our analysis on the various commodity asset pricing models that we investigated: "What has happened is that we've used these assumptions for so long that we've forgotten that we've merely made assumptions, and we've come to believe that the world is necessarily this way." This is the eye of the storm… Just as Jagannathan and Wang (1996) imparted key insight into the CAPM by questioning that model's underlying assumptions, and broadening its underpinnings to include the "aggregate wealth portfolio of all agents in the economy," likewise our thesis questions the suppositions of the various commodity asset pricing models, as well as questions some of the methods which supposedly identify persistent sources of return in the commodity futures markets. Our findings, in turn, contextualize the likely reason for the continuing legacy of inconsistent empirical tests. Unbeknownst to modern finance, the commodity futures markets may be the shoals against which rational expectations equilibrium, the "de facto ruling paradigm of financial economics," is eventually shipwrecked.

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