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**WHAT EVERY INVESTOR SHOULD
KNOW ABOUT COMMODITIES
PART II: MULTIVARIATE RETURN ANALYSIS**

**Harry M. Kat
Roel C.A. Oomen**

**Alternative Investment Research Centre
Cass Business School, City University
106 Bunhill Row, London, EC1Y 8TZ
United Kingdom
Tel. +44.(0)20.70408677
E-mail: harry@airc.info
Website: www.cass.city.ac.uk/airc**

What Every Investor Should Know About Commodities

Part II: Multivariate Return Analysis

Harry M. Kat and Roel C.A. Oomen*

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Please address all correspondence to:

Harry M. Kat
Professor of Risk Management and
Director Alternative Investment Research Centre
Cass Business School, City University
106 Bunhill Row, London, EC1Y 8TZ
United Kingdom
Tel. +44.(0)20.70408677
E-mail: harry@airc.info

*Harry M. Kat is Professor of Risk Management and Director Alternative Investment Research Centre, Cass Business School, City University, London. Roel Oomen is with the Department of Finance, Warwick Business School and is also a research affiliate of the Department of Quantitative Economics at the University of Amsterdam, The Netherlands. The authors like to thank CSI and Rudi Cabral for allowing us to access the CSI commodity database, and Phillip Vitale and Jason Malinowski for providing the bond return series used.

What Every Investor Should Know About Commodities

Part II: Multivariate Return Analysis

Abstract

In this paper we study the multivariate return properties of a large variety of commodity futures. We find that between commodity groupings (such as metals, energy, etc.) correlations are very low and mostly insignificant whereas within groups they tend to be much stronger. In addition, commodity futures are roughly uncorrelated with stocks and bonds. Still, correlations may vary somewhat over the different phases of the business cycle, suggesting that not all commodities make equally good diversifiers at all times. Copula-based tests do not indicate any deviant behaviour in the tails of the joint return distribution of commodity futures and stocks or bonds. Contrary to equities and bonds, we show that commodity futures returns are positively correlated with unexpected inflation (i.e. 25% on average with CPI inflation as opposed to -30% for equities and -50% for bonds). There are significant differences between the various commodities, however, with energy, metals, cattle, and sugar offering the best hedging potential. Altogether, assuming that the observed regularities will persist, our results confirm that a well-balanced commodity futures portfolio could offer a worthwhile diversification service to the typical traditional investment portfolio.

Keywords: Commodities, commodity futures, correlation, tail-dependence, SJC copula, inflation.

JEL Classifications: G11, E44, O13, Q19, Q49.

Introduction

With interest rates at historically low levels, little obvious potential in traditional asset classes, and risk premiums steadily declining, many investors have slowly but surely come to embrace 'alternative' investments, such as hedge funds, private equity and commodities. The optimistic growth prospects of large developing countries like Brazil, China and India, and the accompanying need for oil, industrial metals and construction supplies, have convinced many investors that the only way for commodity prices is up. As a consequence, investment in commodities is growing at an unprecedented rate. On the recommendation of their consultants, even pension funds make substantial allocations to commodities these days. According to a recent institutional investor survey by Barclays Capital, over the next three years 60% of the 150 respondents, 70% of whom currently have no commodity exposure, plan to increase their commodity allocation to 6% or more¹.

In a well-diversified portfolio, the overall portfolio's risk-return characteristics are primarily determined by the relationship between the various asset classes that make up the portfolio. When considering investing in a new asset class, it is therefore important to carefully analyse the relationship between the returns on that particular asset class and the returns on asset classes already present in the portfolio. In addition, when an asset class is very heterogeneous in nature, investors should also carefully study the relationships between the individual assets that make up that asset class, as in those cases a brute force indexation approach ("just buy a little of everything"), as is often used in the equity and bond markets, may not be the preferred way to gain exposure to that particular asset class.

Since commodities have not been very popular as an asset class until recently, little is known about the multivariate properties of commodity futures returns. Two recent papers shed some light on these matters though. Gorton and Rouwenhorst (2006) studied the long-term performance of an equally-weighted index of 36 commodities. They show that over the period 1959 -2004 their index was negatively correlated with stocks and bonds, and positively correlated with inflation. Erb and Harvey (2006) perform a similar analysis and find that sensitivity to inflation varies significantly across different commodities, implying that not all commodities are equally good inflation hedges.

Especially the Gorton and Rouwenhorst (2006) study has attracted a lot of attention in the media as well

¹As reported by Reuters December 16th, 2005

as the asset management and pension fund industry. Since it concentrates on one particular index, and therefore does not account for the high degree of heterogeneity in commodities, it is difficult to gauge the true significance of the authors' findings. Unfortunately, this has not stopped several large pension fund consultants from recommending their clients to make significant allocations to commodities, with the Goldman Sachs Commodity Index (GSCI) being the most popular vehicle. A senior investment consultant at Watson Wyatt in a recent Financial Times article², even went as far as stating that: "Commodities are a relatively easy asset class to understand and to invest in, so they provide low hanging fruit for pension funds' diversification purposes". To make a well-informed investment in commodities and to be able to construct a sensible portfolio, investors need to understand the differences between different (types of) commodities, how they relate to each other as well as how they relate to other asset classes. Knowing how one particular portfolio of commodities behaves is interesting, but insufficient when it comes to allocating billions of dollars to a commodity portfolio that is completely different in composition than the one used in the above study³.

Motivated by the above, this paper provides a comprehensive analysis of the multivariate⁴ return properties of commodity futures. More in particular, we attempt to answer the following five questions:

1. Are commodity futures mutually correlated?
2. Are commodity futures correlated with stocks and bonds?
3. How is the tail-dependence among commodity futures?
4. How is the tail-dependence between commodity futures and stocks and bonds?
5. How are commodity futures related to inflation?

[Insert Table 1 Here]

The database used in this paper is the same as in Kat and Oomen (2006): daily settlement prices on 142 different commodity futures contracts (including different trading locations for the same commodity), covering the period January 1965 – February 2005 (where available). For brevity, as in Kat and Oomen (2006), we only present the results for a representative selection of 27 commodities, and for the nearby

²See Financial Times, FTfm section, March 20, 2006, page 3

³Contrary to the Gorton and Rouwenhorst (2006) index, which contains 14% energy, the GSCI contains 75% energy.

⁴See Kat and Oomen (2006) for the univariate properties

contract only. The commodity futures we will concentrate on, together with their basic marginal sample properties, can be found in table 1.

Question 1: Are Commodity Futures Mutually Correlated?

Co-movement in commodity prices can be attributed to a number of possible causes. First, common macroeconomic shocks may directly affect a number of commodities at the same time. Second, some commodities may complement or substitute for each other in production or consumption. Third, although typically unrelated, some commodities may share a common event factor, which relates them only at specific times or during specific periods. Finally, herd behaviour could also cause otherwise unrelated commodities to become dependent. It is, however, difficult to see how the market, which typically appears to react quite rationally, could allow for such an inefficiency to arise and persist.

To investigate the contemporaneous dependence among commodity futures returns we computed the standard (Pearson's) correlation coefficient, as well as Kendall's Tau statistic. The latter is a rank based measure of dependence that is, unlike the correlation coefficient, robust to non-linear dependence. We briefly review both dependence measures in Appendix A.1.

[Insert Table 2 Here]

Table 2 reports the correlation coefficient in the upper diagonal and Kendall's Tau in the lower diagonal for the daily excess returns on the 27 different commodity futures from table 1, over the longest data sample available. The figures in boldface indicate 5% significance (from zero) based on the bootstrap with 10,000 replications⁵. From the table we see that, overall, commodity futures returns tend to exhibit low dependence across groups, but high dependence within groups. For instance, within energy the lowest correlation is 47% (between gasoline and gas) and the highest correlation is 84% (between gasoil and crude oil). In contrast, the highest correlation between energy and any of the other 23 commodities is only a mere 16%. Similar observations can be made for the results based on Kendall's Tau. This suggests that from an overall short-term risk reduction perspective, diversification across commodity groups is to be preferred over diversification within a given group. The exception to this is the softs category, within which correlations are very low. We will return to this shortly.

⁵See Appendix A.2 for details on the bootstrap procedure used.

Similar to Kat and Oomen (2006), we repeated the above calculations while classifying our results based on ex-post business cycle conditions. For the business cycle we used the usual NBER dating. We also classified our results based on the prevailing monetary regime. Neither classification, however, produced significantly different results, not between, nor within different commodity groups. These results are therefore not reported. To investigate the relationship between different commodity futures over longer horizons, we also repeated the above analysis using monthly instead of daily returns. Again, we found no significant differences in that monthly correlations were of the same order of magnitude as daily correlations.

The above results suggest that the traditional grouping of commodities into grains and oils seeds, softs, meat and livestock, energy and metals may be somewhat misleading, as the commodities typically placed in the softs group exhibit very little mutual correlation. To further investigate the composition and statistical properties of each commodity grouping, we performed a so-called principal component analysis (PCA)⁶. PCA is a data reduction technique, which transforms the original data set into a number of “factors” or “principal components”, with the aim to identify the key sources of variation. These factors are constructed such that they are (i) a linear combination of the original data, (ii) mutually orthogonal and (iii) ordered in terms of explained variation. Thus, the first principal component is the linear combination of the data with maximum variability and can be thought of as capturing the most salient features of the data. The second principal component is independent of the first and incorporates the next most variation, and so on. Intuitively, one can think of the S&P 500 index as a principal component in that it is a linear combination of large cap US stocks (with weights given by market capitalization) and aims to capture the salient features of this asset class. PCA formalizes this idea by choosing the weights optimally so as to absorb the maximum amount of data variation into the principal component.

[Insert Table 3 Here]

Table 3 reports the first eight principal components of the 27 commodities from table 1. A number of observations can be made. First, energy accounts for one-fifth of the variation in the dataset, underlining that energy is a key driver of risk in commodity futures returns. Second, energy, metals, and grains and oil seeds are all identified as separate risk factors. This means that for these categories the traditional grouping coincides with the PCA grouping, which is purely based on statistical criteria. As

⁶See for instance Jolliffe (2002) for more details.

a consequence, however, diversification within these groups will be of little value. Third, the softs category, and to a lesser extent the meat and livestock category as well, is not identified as a separate risk factor, which is in line with the relatively low correlations found within this category in table 2. Investors need to be very careful not to treat these commodities as similar, as they clearly have distinctly different risk characteristics. Given the lack of common ground, however, diversification among these commodities will be more beneficial than diversifying within the categories energy, metals or grains and oil seeds.

Question 2: Are Commodities Correlated with Stocks and Bonds?

Commodities are fundamentally different from financial assets. As a result, there are at least two reasons why it is plausible that commodities will be negatively correlated with stocks and bonds. First, instead of longer-term economic prospects, commodity prices are primarily determined by current economic activity. This means that commodities are likely to be at their best during the expansion phase, and at their worst during the recession phase of the business cycle. Stock and bond performance will tend show an opposite picture though, doing best during the late recession/early expansion and worst during the late expansion/early recession phase. Second, contrary to stocks and bonds, commodities tend to have positive exposure to event shocks. An unexpected cut in the supply of a commodity (for example OPEC cutting crude oil production, crop freezes, or a strike in the mining industry) will drive the commodity's price up, producing a positive return for investors who are long that commodity. Stocks, however, may come under pressure as a result of the increased costs of raw materials.

To investigate the relationship between commodity futures and the stock market, we calculated the correlation coefficient and Kendall's Tau between the daily excess returns on the same 27 commodity futures as before and the daily excess return on the Dow Jones Industrial Average (DJIA). We did so for the whole sample period, as well as for different business cycle phases (SR = start recession, ER = end recession, SE = start expansion, and EE = end expansion) and different monetary environments (EM = expansive, i.e. rate decrease in last 3 months, RM = restrictive, i.e. rate increase in last 3 months, and AM = awaiting, i.e. rate unchanged for at least 3 months). To see whether there is any difference in behaviour between more and less volatile periods, we also categorized our results based on equity market and commodity market volatility. Furthermore, to gauge the correlation dynamics we

fit a so-called GARCH(1,1)-DCC(1,1) model⁷ to the data and report the estimated autoregressive (AR) parameter of the correlation equation. When this parameter is very close to 1, correlation evolves over time in a persistent manner so that extended periods of high and low correlation are common. Finally, we investigate whether the correlation between commodity futures and the stock market is different over different horizons by repeating the above using weekly, monthly and quarterly returns. The results are reported in tables 4 and 5, with figures in boldface indicating 5% significance based on the bootstrap. Note that, apart from the full sample correlations in the first column, all other columns report the difference between correlation over the subsample (e.g. start of recession) and its complement (e.g. all periods except start of recession).

[Insert Tables 4 and 5 and Figure 1 Here]

From tables 4 and 5, a number of interesting observations can be made:

1. Overall, daily commodity futures returns are only weakly correlated with equity returns. Unconditionally, the correlation between equity and grains and oil seeds is only about 5%, whereas with energy it is about -5%. Only the correlation with lumber and copper exceeds 10%. Most other commodities have insignificant correlation with equity. Based on Kendall's Tau statistic, the dependence appears even weaker.
2. The GARCH-DCC estimation results (see DCC column) indicate there is little persistence in correlation dynamics. The AR parameter averages 0.68, which corresponds to a half-life of less than 2 days. Even for energy, where the DCC parameter is close to 0.95, the time series of equity-correlations appears to exhibit little persistence. This is illustrated in panel A of figure 1, which plots the GARCH-DCC-filtered correlation between crude oil and equity. Metals are a notable exception, however, with DCC parameters of 0.99 for each constituent. The solid line in panel B of figure 1 plots the GARCH-DCC-filtered correlation between gold and equity. Clearly, there is considerable persistence in the correlation dynamics, featuring extended periods of high correlation (e.g. mid-80's) and low negative correlation (e.g. 1988, 1991, 2003). Panels C and D plot the evolution of crude oil and gold volatility and confirm that these variations cannot be explained by shifts in volatility.

⁷See Engle (2001) for details on the DCC model. To estimate this model we used the UCSD_Garch matlab toolbox of Kevin Sheppard, which is available at www.kevinsheppard.com.

3. Correlation with equity can vary over the different phases of the business cycle, particularly for energy and metals. During the end of an expansion the correlations for energy increase significantly by about 8%, while for metals they significantly decrease by about 15%. On the other hand, at the start of a recession the correlation with energy is reduced very substantially, yielding a significantly negative correlation in such environments. For metals we observe the reverse, with significantly higher correlations during recessions. With Kendall's Tau these patterns remain, although they are less strong for energy.
4. The classification based on monetary regime or equity market volatility does not reveal any systematic variation in the equity-commodity dependence.
5. When commodity markets are volatile, the equity-correlations tend to increase for metals and decrease for energy and to a lesser extent grains and oil seeds. The opposite pattern is observed when commodity markets are tranquil. The results based on Kendall's Tau are similar albeit less significant.
6. With the exception of metals, increasing the differencing interval appears to reduce the correlation between commodity futures and equity. However, this effect is hardly significant and not observed in Kendall's Tau. This suggests that the observed drop in correlation is primarily the result of a limited number of extreme observations. It also casts doubt on the conclusion of Gorton and Rouwenhorst (2006) that the diversification benefits of commodity futures tend to be larger at longer horizons.

Altogether, the above results confirm that the correlation between commodity futures and equity is weak and sometimes even negative. However, they also show that not all commodities are equal when it comes to diversification potential, nor that they all work at the same time. Energy makes a particularly good diversifier during recessions. During the end of an expansion, however, metals, meat and livestock and softs are likely to be more effective.

[Insert Tables 6 and 7 Here]

Tables 6 and 7 provide the same information as found in tables 4 and 5, but this time for bonds instead of equity. Bonds are taken to be US Treasury bonds with a maturity of 10 years. From the tables we see that the correlation between commodity futures and bond returns is very weak. Looking at the different phases of the business cycle, we see that correlation tends to be relatively low during

the end of an expansion, and relatively high during a recession. As is often the case, energy shows the exact opposite behaviour though. Metals show quite interesting behaviour as well, exhibiting relatively strong positive correlation during the end of a recession. Contrary to what we saw before in tables 4 and 5, there is a tendency for the correlation between commodity futures and bond returns to increase when equity markets become more volatile. Commodity market volatility on the other hand tends to have the opposite effect, especially in energy. Metals are once again the exception, registering an increase in correlation when market volatility rises. All in all, however, commodity correlation with bond returns is generally very low and insignificant.

Question 3: How is the Tail-Dependence Among Commodities?

Correlation is without doubt the single most important parameter in portfolio theory, where it is used to measure the dependence between the returns on different assets or asset classes. The rule is simple: low correlation makes for good diversification and highly correlated assets or asset classes are to be avoided. Half a century after Markowitz this way of thinking has become so common that nowadays most people use the terms “correlation” and “dependence” interchangeably. When dealing with the normal distributions that portfolio theory is based on there is nothing wrong with this. Unfortunately, however, the returns on many alternative assets and asset classes are not normally distributed. In that case, correlation may not be a very good measure of dependence and may actually be quite misleading at times.

When it comes to correlation and dependence the big question is whether the correlation coefficient is sufficient to describe the dependence structure between two returns. Although beyond the scope of this paper, it can be shown that this is only the case when the joint distribution of both returns is elliptical. Elliptical simply means that when the joint distribution is viewed from above, the contour lines of the distribution look like ellipses. The best-known elliptical distribution is of course the normal distribution. Every bivariate normal distribution can be fully described by just two expectations, two variances and one correlation coefficient.

[Insert Figure 2 Here]

Unfortunately, elliptical distributions are nothing more than an ideal type that is rarely encountered in practice. Elliptical distributions, however, are also the easiest distributions to work with mathematically. As a result, the assumption of normality has become the single most important assumption

in econometrics, which has left us in the awkward situation where the bulk of the econometric tools we have at our disposal assume a distribution that is hardly ever observed in reality. As said, if the distribution is not elliptical, the correlation coefficient may not be a good measure for the dependence structure between the two returns involved. An example is provided in figure 2, which shows plots from a three different bivariate distributions. These plots look quite different, implying a completely different dependence structure. However, they all have a correlation coefficient of 0.7.

In non-normal distributions the so-called “tail dependence” can be quite pronounced as there may be a tendency for extreme values of both variables to occur simultaneously. This illustrates an important point: if two variables are both normally distributed this does not necessarily mean that their joint distribution is normal as well. This is only the case if we assume that the joint distribution is elliptical. If not, there are an infinite number of bivariate distributions that fit this description. Another point concerns the values that the correlation coefficient can take. We all know that because the correlation coefficient equals the normalized covariance, it will always lie between $+1$ and -1 . However, whether it is actually possible for the correlation coefficient to take on these extreme values is another matter. For non-elliptical distributions the attainable interval might well be smaller. For some distributions the attainable interval can be very small, say between -0.4 and $+0.4$. If this is indeed the case, finding a correlation coefficient of 0.4 and concluding that there was only weak dependence between both variables involved would be a mistake as both are actually perfectly dependent.

[Insert Figure 3 Here]

Given the above, we proceed with an explicit analysis of the tail-dependence characteristics of commodity futures returns. We first look at the tail-dependence among commodity futures and then proceed with the tail-dependence between commodity futures and stocks and bonds. Figure 3 shows a number of scatter plots of the pair-wise weekly returns on the various commodities classified under 'Energy'. All distributions appear elliptical, without any indication of abnormal behaviour in the tails. Similar results were obtained for other commodities (and therefore not reported).

To confirm the above conclusion formally, we estimated a so-called “Symmetrised Joe-Clayton” (SJC) copula, developed by Patton (2006). Intuitively, a copula divides the characteristics of the bivariate distribution between two variables into two marginal distributions, which contain the univariate characteristics of each variable, and the copula, which contains all information concerning the dependence

between these random variables. There are various copulas around these days⁸. We decided to use the SJC because it is one of the most flexible copulas available with two parameters (referred to as τ_L and τ_U), which separately measure the dependence in the lower and upper tail. As a result, this copula can fit data with very different patterns of dependence in the tails. More details can be found in Appendix A.3.

[Insert Table 8 Here]

Table 8 shows the estimates for the upper (upper diagonal) and lower (lower diagonal) dependence parameters of the SJC copula. With the exception of softs and in line with what we found before in table 2, we see quite some dependence between commodities within the same group, but very little between commodities belonging to different groups. Comparing upper and lower dependence parameters, it appears that for none of the commodity pairs studied there is a big difference in dependence between the lower and upper tail. We repeated the above using monthly instead of daily returns, with very similar results (not reported). Thus, it seems safe to conclude that the joint distribution of commodity futures returns is reasonably well behaved and that the standard correlation measures provide useful and reliable information with regard to the dependence structure of returns.

Question 4: How Is the Tail-Dependence Between Commodities and Stocks and Bonds?

Knowing that there is no deviant behaviour in the tails of commodity futures return distributions, the next question of course is whether this is also true for the relationship between commodity futures and stocks and bonds. To investigate this, we first plotted the weekly returns for stocks (DJIA) and bonds (10-year US Treasury) versus the weekly returns on commodity futures. The results for crude oil and gold can be found in figure 4. As for oil and gold, none of the scatterplots suggest abnormal tail behaviour. In fact, at the time of the '87 stock market crash, bonds, gold, as well as oil all recorded positive returns.

[Insert Figure 4 Here]

Next, we checked in more detail whether extreme events in different markets have a tendency to occur simultaneously. Concentrating on the average of the 10 most extreme returns in commodity futures, stocks and bonds, the result can be found in table 9.

⁸See for example Joe (1997) for a review of copula theory

[Insert Table 9 Here]

The first two columns in table 9 show for every commodity the average of the 10 lowest and 10 highest daily returns over the period 1987–2005. This confirms that commodity futures are capable of generating quite extreme returns at times⁹. The next four columns show bond and equity returns at the time of extreme commodity futures returns. From here it is clear that there is no relationship between extreme commodity returns and bond and equity returns. The same conclusion follows from the last four columns of table 9, which shows commodity futures returns at the time of extreme bond and equity returns. Overall, there is no evidence of abnormal tail behaviour in the relationship between commodity futures, bond and equity returns.

[Insert Table 10 Here]

To confirm the above conclusion formally, we again estimated the SJC copula as before, on daily as well as weekly, monthly and quarterly returns. The estimated tail dependence parameters can be found in table 10. Given the low overall correlations found earlier, the table entries show no indication of abnormal behaviour in the tails, which confirms the results in table 9.

Question 5: How Are Commodities Related to Inflation?

Bottom line, investors don't just want to make money. They want to gain purchasing power, i.e. outperform inflation¹⁰. Because, unlike stocks and bonds, commodity prices are not discounted future cash flows, it stands to reason that the relationship between commodity returns and inflation is fundamentally different as well. In times of strong economic growth, there will be upward pressure on commodities, producer and consumer prices, as well as interest rates. Higher commodity prices and higher interest rates reduce the growth potential of company profits and at the same time reduce the present value of future cash flows. As a result, stock and bond returns will drop, but commodities will still be going strong. From this perspective, higher inflation is likely to have a negative impact on stock and bond returns, but a positive impact on commodities. Of course, this makes sense if one realizes

⁹For comparison, over the period 1987-2005 the average of the 10 largest negative (positive) daily DJIA returns was -8.52% (5.51%), which is in line with the finding of Kat and Oomen (2006) that commodity futures are not more volatile than US large cap stocks.

¹⁰This is especially important for institutional investors as many of them have liabilities that are explicitly indexed to inflation in some way.

that, directly and/or indirectly, commodity prices are an important component of the price indices by which inflation is typically measured.

When it comes to measuring inflation, there is a large number of price indices to choose from. Since different types of inflation are relevant for different economic agents, we use not one, but three different US inflation indices:

1. CPI – Consumer Price Index (with and without food and energy)
2. PPI – Producer Price Index (finished goods with and without food and energy)
3. ECI – Employment Costs Index (public and private)

The CPI measures the price level of a basket of consumer goods. The basket is reset every 2 years, based on consumer behaviour over the 2 years before. Apart from a whole range of sub-indices, two different types of indices are published. The CPI-U concentrates on the whole population in and around urban conurbations, while the CPI-W concentrates on wage earners and clerical workers only. The CPI-U covers 87% and the CPI-W 32% of the US population. We therefore use the CPI-U (which for simplicity we will refer to as CPI). The PPI measures producer prices, based on fixed weighings that are aligned to actual production conditions at regular intervals. Producer prices are sometimes seen as a leading indicator for consumer prices. However, as the CPI reflects prices paid by individuals for consumption, while the PPI reflects prices paid by companies for production, a number of factors, including taxes, profit margins and wholesale and retail costs, may distort this relationship. In addition, different survey methods are used for both indices. Finally, the ECI measures the total remuneration (excl. stock options) of employees in the private and/or public sector. In total 90% of US employees are covered. Unlike the CPI and PPI, which are published on a monthly basis, the ECI is published quarterly. We use CPI and PPI data from 1973 and ECI data from 1982, at an annual frequency.

Since expected future inflation will already be incorporated in asset prices, what really matters is not how commodity futures respond to inflation, but how they respond to unexpected inflation. Unexpected inflation is not easy to measure, however. One possibility is to estimate unexpected inflation from T-bills, i.e. to calculate unexpected inflation as the difference between actual inflation and the interest rate¹¹. Another alternative is to use the change in the inflation rate as a proxy for unexpected

¹¹See for example Fama and Schwert (1977)

inflation, implicitly assuming that today's inflation is the best predictor of future inflation as implied by the random walk model. In this paper we use the latter approach. Specifically, let P_t denote the CPI/PPI/ECI price index at the end of year t . The inflation rate is then defined as $I_t = (P_t - P_{t-1})/P_{t-1}$ and unexpected inflation is equal to $i_t = I_t - I_{t-1}$.

[Insert Table 11 Here]

Table 11 shows the correlations between annual commodity futures returns and unexpected inflation¹². To gain insight into the actual source of the correlation, futures returns (Ftr) are split up between spot return (Spt) and roll return (Rll). From the table we see that, overall, commodity futures returns and unexpected inflation are positively correlated. The average correlation with the CPI is 25.1%, with the PPI 23.3% and 22.8% with the ECI (private). Unlike stocks and bonds (see bottom rows), commodity futures therefore tend to provide investors with a hedge against unexpected inflation. There are significant differences between different commodities though. Energy, metals, cattle, and sugar offer the best hedging potential. Grains and oil seeds, softs (excl. sugar), pork and palladium on the other hand have little or even a negative relationship with unexpected inflation. Apart from substantially lower correlations for the ECI (public), there does not seem to be much difference between the main inflation measures. Not surprisingly, taking food and energy out of the CPI or PPI index strongly reduces the correlation of energy, meat and livestock, and some softs. It does not seem to harm the correlations of grains and oil seeds though. Finally, the division between spot and roll return suggests that commodity futures' correlation with unexpected inflation primarily stems from the spot market. Correlation of roll returns with unexpected inflation is largely insignificant. This finding contradicts Erb and Harvey (2006), who find a much more important role for roll returns.

Conclusion

In this paper we have shed some light on the multivariate return characteristics of a large variety of commodity futures. The answers to the questions asked in the introduction are as follows:

- A1.** Correlations among commodities in different groupings are small and largely insignificant. Correlations within commodity groupings are substantially higher, with the exception of softs, which appears to be made up of commodities that have little in common.

¹²Note that because of the annual frequency, the number of observations is quite limited.

- A2. Overall, commodity futures are uncorrelated with stocks and bonds. Correlation may vary somewhat between different phases of the business cycle, however, which suggests that not all commodities make equally good diversifiers at all times.
- A3. There is no indication of deviant tail behaviour within or across commodity groupings.
- A4. There is no indication of deviant behaviour in the tails of the joint return distribution of commodity futures and stocks or bonds. Put simply, commodities do not lose their diversification properties in a turbulent market environment.
- A5. Overall, commodity futures and unexpected inflation are positively correlated. There are significant differences between different commodities though, with energy, metals, cattle, and sugar offering the best hedging potential.

Given the above conclusions, (a portfolio of) commodity futures could provide a significant diversification service when incorporated in a traditional investment portfolio. However, it is important to remember that, unless one believes that “this time it will all be different”, this diversification service does not come for free. As shown in table 1, most commodity futures do not offer investors a positive risk premium. Adding commodity futures to a portfolio will therefore not only reduce risk, but also expected return. The only exception is energy, which does appear to offer a risk premium. Heavily weighting a commodity futures portfolio towards energy will make that portfolio substantially more risky though, as energy is more volatile by itself (see table 1) and different types of energy commodity futures are strongly correlated (see table 2). On the other hand, with more energy the commodity portfolio’s correlation with stocks and bonds will come down somewhat (see table 4–7), which in an overall portfolio context will serve to dampen part of the increase in volatility due to the higher energy concentration. *Ceteris paribus*, a portfolio with a low energy allocation can be expected to have lower volatility, but also a lower expected return and vice versa. In the end, whether to overweigh energy (by investing in the GSCI for example) is therefore very much a matter of taste, and not necessarily the golden rule.

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A Appendix

A.1 Correlation measures

Pearson's correlation coefficient for variables x and y is computed as:

$$\rho = \frac{1}{N-1} \sum_{i=1}^N \frac{(x_i - \hat{\mu}_x)(y_i - \hat{\mu}_y)}{\hat{\sigma}_x \hat{\sigma}_y} \quad (1)$$

where $\hat{\mu}_x = \frac{1}{N} \sum_{i=1}^N x_i$ and $\hat{\sigma}_x^2 = \frac{1}{N-1} \sum_{i=1}^N (x_i - \hat{\mu}_x)^2$. The correlation coefficient ρ measures the degree of *linear* dependence between x and y , that is, in the linear regression $y = \alpha + \beta x$ the regression R^2 is equal to the squared correlation between x and y . Clearly, if the relationship between x and y is not linear (e.g. $y = \alpha + \beta x^3 + \varepsilon$) then the R^2 , and thus the correlation coefficient, does not provide a reliable measure of dependence between the two variables. In that case, the *rank based* Kendall's Tau statistic will be more appropriate. In a sample of N observation, there are $\binom{N}{2} = N(N-1)/2$ possible pairings of (x_i, y_i) and (x_j, y_j) . The pairing is said to be concordant if $\{x_i > x_j \cap y_i > y_j\}$ or $\{x_i < x_j \cap y_i < y_j\}$ and discordant if $\{x_i < x_j \cap y_i > y_j\}$ or $\{x_i > x_j \cap y_i < y_j\}$. If $\{x_i = x_j \cup y_i = y_j\}$ the pairing is "tied" and these are neither concordant or discordant. Kendall's Tau is then defined as:

$$\tau = \frac{n_c - n_d}{\sqrt{((\binom{N}{2}) - n_x)((\binom{N}{2}) - n_y)}}$$

where n_c (n_d) is the number of concordant (discordant) pairings and n_x (n_y) is the number of ties in x (y). Thus, intuitively, Kendall's Tau can be interpreted as the probability of observing a concordant pair minus the probability of a discordant pair. An important property of Kendall's Tau is that it is invariant under monotone transformations. For instance, Kendall's Tau between x and y , $\log(y)$, $\exp(y)$ or y^3 are all identical. In this paper we construct an approximation to Kendall's Tau as:

$$\tau_{\text{approx}} = \frac{1}{N} \sum_{i=1}^N 1_{\{x_i > 0 \cap y_i > 0\}} + 1_{\{x_i < 0 \cap y_i < 0\}} - 1_{\{x_i > 0 \cap y_i < 0\}} - 1_{\{x_i < 0 \cap y_i > 0\}} \quad (2)$$

The main motivation for using this approximation is that it is much faster to Compute. Bootstrapped significance levels are therefore straightforward to obtain.

A.2 Bootstrapped confidence intervals

Here we briefly outline how we obtain significance levels using the bootstrap. Consider two data samples $Z_1 = \{x_1, y_1, \dots, x_{N_1}, y_{N_1}\}$ and $Z_2 = \{x_{N_1+1}, y_{N_1+1}, \dots, x_{N_1+N_2}, y_{N_1+N_2}\}$. Suppose we compute

Kendall's Tau for both these samples, i.e. $\hat{\tau}_1$ and $\hat{\tau}_2$, and want to test whether their difference is significantly different from zero, i.e. $H_0 : \hat{\tau}_1 \neq \hat{\tau}_2$. The bootstrap method resamples with replacement from the original data Z_1 and Z_2 to construct N_B bootstrap samples of equal dimension, i.e. $\{Z_1^{(i)}, Z_2^{(i)}\}_{i=1}^{N_B}$. For each bootstrap sample we then calculate Kendall's Tau to obtain $\{\hat{\tau}_1^{(i)}, \hat{\tau}_2^{(i)}\}_{i=1}^{N_B}$. If $\hat{\tau}_1 > \hat{\tau}_2$, then the associated bootstrapped p-value is calculated as:

$$\frac{1}{N_B} \sum_{i=1}^{N_B} 1_{\{\hat{\tau}_1^{(i)} > \hat{\tau}_2^{(i)}\}}$$

and vice versa.

A.3 The Symmetrized Joe-Clayton Copula

The joint distribution of x and y can be fully characterized by the marginal distributions $F_x(x)$ and $F_y(y)$ plus the corresponding copula function $C(F_x(x), F_y(y))$. In this paper we use the symmetrized version of the Joe-Clayton copula (See Patton (2006)) to measure tail-dependence.

$$C_{SJC}(u, v|\tau) = \frac{1}{2} \{C_{JC}(u, v|\tau) + C_{JC}(1-u, 1-v|\tau) + u + v - 1\} \quad (3)$$

where $\tau = \{\tau^U, \tau^L\}$, $u = F_x(x)$, $v = F_y(y)$, $\kappa = 1/\log_2(2 - \tau^U)$, $\gamma = -1/\log_2 \tau^L$, $c_u = 1 - (1-u)^\kappa$, and $c_v = 1 - (1-v)^\kappa$ and

$$C_{JC}(u, v|\tau^U, \tau^L) = 1 - \left(1 - (c_u^{-\gamma} + c_v^{-\gamma} - 1)^{-1/\gamma}\right)^{1/\kappa}$$

The parameters τ^U and τ^L have a natural interpretation, namely that of upper- and lower-tail dependence, i.e.

$$\begin{aligned} \tau^L &= \lim_{\varepsilon \rightarrow 0} \Pr[u \leq \varepsilon | v \leq \varepsilon] = \lim_{\varepsilon \rightarrow 0} \Pr[v \leq \varepsilon | u \leq \varepsilon] \\ \tau^U &= \lim_{\delta \rightarrow 1} \Pr[u > \delta | v > \delta] = \lim_{\delta \rightarrow 1} \Pr[v > \delta | u > \delta] \end{aligned}$$

For given data (x, y) , the log-likelihood is calculated as $\mathcal{L}(\tau) = \sum_{i=1}^N \log c_{SJC}(u_i, v_i|\tau)$ where the copula density $c_{SJC}(\cdot) = \partial^2 C_{SJC}(\cdot)/\partial u \partial v$. Maximum likelihood estimates (MLE) of τ are obtained by maximizing $\mathcal{L}(\tau)$ over the parameter space using numerical methods. To avoid potential misspecification of the marginal distribution, in this paper we simply use the empirical cdf to back out u and v . This reduces the numerical instability one often encounters with parametric distributions where u and v can be very close to their boundaries 0 or 1. A formal justification for this approach can be found in Genest, Ghoudi, and Rivest (1995).

Table 1: Summary statistics for daily commodity futures returns in USD

	Start (# obs)	moments				quantiles				serial correlation		
		mean	volatility	skewness	kurtosis	min	2.5%	97.5%	max	ρ_1	ρ_2	ρ_3
<i>Category I: Grains and oil seeds</i>												
1. Corn	1965 (9904)	-5.87**	19.8	0.05	5.42**	-8.79	-2.74	2.76	6.98	6.37**	-2.10**	-0.32
2. Oats	1965 (9904)	-5.73*	27.7	-0.04	4.98**	-11.0	-3.76	3.67	11.1	5.31**	-3.13**	1.61*
3. Soybeans	1965 (9902)	-0.30	22.4	-0.22*	7.09**	-15.3	-3.19	2.99	12.5	5.71**	-0.01	1.74**
4. Wheat	1965 (9904)	-5.39*	23.0	0.10**	4.97**	-9.51	-3.06	3.08	9.08	2.36**	-4.06**	0.25
5. Flaxseed	1970 (8472)	0.54	21.1	0.13*	5.52**	-7.34	-2.79	2.88	9.38	13.1**	0.50	-0.17
6. Rapeseed	1970 (8520)	-1.12	21.3	0.17	6.14**	-9.30	-2.85	2.88	15.9	7.06**	-3.03**	1.99**
<i>Category II: Softs</i>												
7. Azuki Beans	1984 (4995)	-0.89	25.6	-0.01	3.82**	-6.30	-3.38	3.23	10.1	13.2**	5.66**	-0.76
8. Rough Rice	1987 (4566)	-10.6**	24.3	0.08	5.72**	-9.10	-3.24	3.26	9.26	15.5**	-0.95	3.18**
9. Cocoa	1966 (9755)	1.65	29.9	0.12**	4.37**	-10.0	-3.74	3.72	12.6	4.02**	-2.59**	2.60**
10. Sugar	1966 (9644)	-8.32	41.4	-0.24**	6.40**	-23.4	-5.46	5.22	14.2	0.80	-4.09**	3.43**
11. Orange Juice	1967 (9532)	0.03	27.9	0.58**	14.3**	-13.5	-3.68	3.65	23.9	4.08**	-2.34**	2.75**
12. Coffee	1972 (8042)	1.64	35.7	0.28	10.5**	-15.0	-4.50	4.52	23.8	2.67**	-2.11**	2.54**
13. Cotton	1967 (9471)	-0.32	22.7	0.02	5.42**	-8.64	-2.98	2.89	11.3	5.10**	-5.00**	-0.26
14. Lumber	1970 (8827)	-3.32	26.0	0.02	2.66**	-4.55	-3.20	3.20	5.05	10.3**	-1.95**	1.99**
<i>Category III: Meat and livestock</i>												
15. Pork Bellies	1965 (9911)	-3.52	33.0	-0.00	2.91**	-7.60	-3.96	3.96	8.32	6.65**	1.62*	1.17
16. Cattle Live	1966 (9848)	4.95**	16.3	-0.12**	4.01**	-6.36	-2.25	2.17	4.26	5.29**	1.81**	3.45**
17. Hogs	1966 (9784)	5.79*	23.4	-0.09**	3.99**	-6.88	-3.20	3.02	6.88	2.41**	-1.46*	2.35**
18. Cattle Feeder	1971 (8361)	2.01	15.8	-0.13**	4.16**	-6.01	-2.10	2.04	3.87	9.28**	2.17**	4.37**
<i>Category IV: Energy</i>												
19. Gasoil	1979 (6383)	12.2**	33.9	-1.03	24.4**	-39.1	-4.32	4.43	14.0	1.03	-2.47**	-3.03**
20. Crude Oil	1983 (5472)	11.8*	36.2	-1.22**	25.7**	-40.0	-4.37	4.53	14.0	3.39**	-4.29**	-2.95**
21. Gasoline	1985 (4981)	18.1**	35.8	-0.66**	12.9**	-31.0	-4.52	4.47	12.3	4.29**	-0.40	-3.64**
22. Gas	1987 (4356)	16.3**	35.2	-1.38	38.3**	-38.9	-4.14	4.10	23.1	14.2**	2.17*	0.48
<i>Category V: Metals</i>												
23. Silver	1967 (9363)	-4.41	28.1	-0.38**	7.10**	-19.4	-3.89	3.62	9.24	4.96**	3.73**	4.48**
24. Gold	1975 (7552)	-3.36	19.3	-0.04	10.6**	-9.91	-2.72	2.42	9.74	-1.86*	3.03**	1.80*
25. Palladium	1977 (7039)	1.10	31.7	-0.11	8.73**	-18.9	-4.32	4.01	15.3	9.07**	0.81	-1.57*
26. Platinum	1965 (9847)	2.00	26.6	-0.23**	6.60**	-16.8	-3.80	3.49	10.8	2.74**	1.78**	1.70**
27. Copper	1966 (9777)	3.36	25.4	-0.20**	5.81**	-12.5	-3.37	3.27	9.14	0.45	-2.73**	2.12**

Note. Summary statistics for nearby futures returns based on all available data up to March 2005. The non-USD denominated contracts, i.e. Flaxseed (CAD), Rapeseed (CAD), and Azuki Beans (JPY), are converted to USD. Mean and volatility are reported in annualized percentages. One (two) asterisk indicates 10% (5%) significance levels (using the bootstrap with 10,000 replications for the mean, skew, and kurt, and the asymptotic Bartlett level for the correlation coefficients).

Table 2: Correlation / Kendall's Tau matrix of commodity returns (daily)

	Grains and oil seeds						Softs						Meat and livestock						Energy						Metals					
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27			
1	-	63	65	59	41	53	5	17	7	9	4	5	16	5	11	9	11	-2	5	4	3	2	15	11	9	12	10			
2		-	53	51	36	43	2	15	9	8	4	5	13	5	10	9	11	2	2	2	2	1	17	12	9	13	11			
3			-	46	48	66	5	19	9	11	3	6	18	5	13	11	13	2	5	3	1	1	20	15	11	15	12			
4				-	31	38	2	16	6	10	5	5	14	6	11	9	11	3	3	2	1	0	15	11	8	11	10			
5					-	62	3	7	7	8	3	6	10	4	9	10	9	5	5	3	2	1	16	13	10	14	13			
6						-	4	15	9	9	4	5	14	6	10	9	10	3	5	3	1	1	17	14	9	13	12			
7							-	-1	3	2	0	2	1	-3	1	1	1	-1	0	-1	-1	-1	7	8	7	8	3			
8								-	2	3	0	1	5	2	2	2	3	-2	0	1	-1	-1	3	0	3	4	3			
9									-	10	4	11	6	3	3	3	2	2	2	1	3	-1	14	12	7	11	8			
10										-	4	7	7	4	5	2	3	3	2	0	0	0	12	11	9	11	10			
11											-	2	6	1	4	5	5	4	4	2	1	4	5	3	2	4	3			
12												-	3	2	0	2	0	1	0	0	0	-2	7	5	4	7	5			
13													-	5	9	6	8	3	2	3	3	0	9	6	4	7	8			
14														-	4	6	5	7	0	0	1	-1	6	5	3	6	9			
15															-	31	65	29	2	1	1	0	7	4	5	6	5			
16																-	44	78	2	3	1	2	7	5	6	6	6			
17																	-	42	2	3	1	1	6	5	3	5	5			
18																		-	-1	1	-2	0	5	3	5	4	7			
19																			-	84	79	55	10	13	4	8	4			
20																				-	66	-	81	52	12	16	4	9		
21																					-	54	57	-	47	10	15	3	7	
22																						-	31	30	26	-	9	16	2	6
23																														
24																														
25																														
26																														
27																														

Note. This table reports the Pearson's linear correlation coefficient in the upper diagonal and the (approximate) Kendall's Tau in the lower diagonal. Boldface entries are significantly different from zero at a 5% confidence level based on the bootstrap.

Table 3: First 8 principle component loadings for commodity returns (1987 – 2005)

Explained variation:	20%	29%	37%	45%	51%	56%	61%	66%
Factor interpretation:	<i>Energy</i>	<i>Grain</i>	<i>Coffee</i>	<i>Pork</i>	<i>Metals</i>	<i>Sugar</i>	<i>Cocoa</i>	<i>OJ</i>
<i>Category I: Grains and oil seeds</i>								
1. Corn	2.15	-30.1	10.4	-21.6	9.83	-6.10	4.38	-0.03
2. Oats	1.41	-45.2	14.8	-33.7	13.5	-15.4	-0.96	-0.14
3. Soybeans	1.79	-30.7	9.99	-19.8	8.46	-4.29	3.95	-1.09
4. Wheat	1.26	-27.7	8.49	-19.6	8.14	-4.56	3.85	2.09
5. Flaxseed	0.28	-17.0	5.07	-11.2	4.17	-3.75	0.66	-0.96
6. Rapeseed	0.48	-21.7	7.96	-14.3	7.65	-3.40	1.87	-0.31
<i>Category II: Softs</i>								
7. Azuki Beans	-0.55	-1.85	0.50	-0.75	-0.19	0.25	2.54	-1.08
8. Rough Rice	0.50	-13.2	5.02	-9.01	2.03	-3.05	2.49	-0.54
9. Cocoa	1.11	-18.9	-12.5	7.79	-0.99	19.1	-88.1	-30.1
10. Sugar	1.43	-23.3	1.47	6.65	0.85	93.2	24.7	2.11
11. Orange Juice	0.52	-5.14	-2.01	2.87	-3.50	3.42	-30.2	92.9
12. Coffee	-0.20	-40.3	-77.9	36.2	19.1	-16.6	18.4	2.39
13. Cotton	1.38	-10.4	5.50	0.62	2.95	2.65	-5.10	6.23
14. Lumber	-0.71	-2.85	2.78	0.35	5.29	0.31	-8.52	-1.14
<i>Category III: Meat and livestock</i>								
15. Pork Bellies	1.06	-23.7	49.0	66.7	10.4	-12.6	3.02	-1.95
16. Cattle Live	0.63	-3.15	5.78	8.42	1.23	-2.04	-1.05	1.11
17. Hogs	0.98	-12.1	25.7	33.7	8.04	-6.08	0.49	0.10
18. Cattle Feeder	-0.05	1.92	3.79	10.2	-0.13	-1.05	-0.36	0.80
<i>Category IV: Energy</i>								
19. Gasoil	53.5	2.84	0.02	-0.07	3.83	0.91	-0.44	-1.03
20. Oil (Crude)	54.6	2.37	-1.57	0.48	3.69	-0.97	0.14	-3.04
21. Gasoline	50.8	3.40	-2.18	0.48	3.49	-0.64	-4.09	-7.41
22. Gas	38.4	3.39	1.02	-1.01	-1.32	-0.35	7.57	15.2
<i>Category V: Metals</i>								
23. Silver	5.98	-18.3	-0.15	3.44	-36.7	-1.92	-3.54	-0.35
24. Gold	4.87	-7.54	-0.59	0.67	-18.4	-0.45	-2.23	-1.34
25. Palladium	1.91	-17.4	-1.54	3.66	-73.3	-9.02	10.3	-4.36
26. Platinum	3.61	-13.6	-1.53	1.74	-39.6	-2.91	1.73	-2.49
27. Copper	1.76	-9.64	0.26	1.01	-15.8	1.32	-1.41	8.79

Note. This table reports the factor loadings of the first 8 principal components for commodity futures returns over the period 1987 – 2005. The figures in boldface highlight the main drivers for each principle component (and do not necessarily indicate statistical significance).

Table 4: Pearson's Linear Correlation with DJIA

	ρ	Business cycle					Monetary regime					Equity vol				Commodity vol			Frequency		
		SR	ER	SE	EE	EM	RM	AM	Low	Med	High	Low	Med	High	Low	Med	High	1W	1M	3M	
		DCC																			
<i>Category I: Grains and oil seeds</i>																					
1. Corn	3.78	0.34	-6.72	7.46	0.56	-1.02	4.52	-2.75	-0.42	1.50	-3.14	1.69	2.58	4.79	-5.54	-1.04	-1.92	-10.5			
2. Oats	3.82	0.99	-2.52	1.46	-0.70	0.99	-2.94	-0.32	2.12	1.54	-0.36	-0.57	4.24	-0.83	-2.00	-0.37	-1.23	-11.3			
3. Soybeans	4.31	0.98	0.50	0.32	4.03	-4.01	4.08	-2.06	-0.46	-1.71	1.08	0.46	8.35	0.08	-5.34	0.15	-2.31	-12.1			
4. Wheat	3.53	0.77	-4.46	1.40	0.48	0.56	-0.19	-2.89	2.57	-0.28	0.33	0.04	8.26	0.52	-6.23	0.55	0.07	-4.78			
5. Flaxseed	3.00	0.74	-5.10	1.48	2.15	-0.78	-2.20	-0.93	2.48	-0.32	-3.54	3.35	4.07	1.48	-3.78	1.76	-1.61	-15.0			
6. Rapeseed	4.14	0.99	-2.90	6.48	3.56	-4.50	3.72	-4.05	1.21	-1.66	-0.89	2.09	0.26	2.13	-1.71	0.08	-7.22	-14.9			
<i>Category II: Softs</i>																					
7. Azuki Beans	-4.98	0.98	22.1	-3.38	-0.82	-3.57	4.46	-5.69	0.97	-0.56	5.90	-5.05	-0.29	0.36	-0.41	-1.01	-5.30	-2.98			
8. Rough Rice	2.45	0.00	-6.32	16.6	0.64	-1.81	6.29	2.80	-5.85	1.52	1.85	-2.29	1.98	-5.72	3.68	0.20	-12.9	14.0			
9. Cocoa	1.59	0.91	6.40	2.54	2.10	-5.74	2.00	-0.99	-0.67	1.34	2.87	-3.19	-0.67	-2.88	3.27	2.28	-11.4	-23.3			
10. Sugar	2.38	0.97	10.5	2.59	-1.98	-3.38	5.06	2.88	-5.71	0.08	2.64	-2.14	-5.03	1.93	1.88	-0.56	-6.25	-14.9			
11. Orange Juice	1.55	0.00	0.02	3.72	0.70	-2.13	1.48	2.59	-3.08	0.25	-0.28	0.22	2.39	-1.35	-0.52	-2.23	-12.0	-13.0			
12. Coffee	1.40	0.24	-0.04	-3.32	2.84	-1.53	1.33	0.61	-1.23	0.29	-0.83	0.62	0.94	-0.41	-0.15	-2.88	3.22	-3.82			
13. Cotton	4.97	0.00	2.01	12.4	-3.43	-2.10	0.15	2.51	-1.95	-1.99	-6.66	6.83	2.28	-1.31	-0.11	1.22	5.54	14.6			
14. Lumber	11.8	0.99	11.9	9.74	1.59	-9.85	5.21	-5.33	0.26	-5.02	-0.87	5.00	-2.33	-0.26	2.96	2.11	11.3	15.8			
<i>Category III: Meat and livestock</i>																					
15. Pork Bellies	0.35	0.17	0.22	3.29	5.36	-6.37	-0.48	-0.94	0.85	1.98	-3.07	1.25	2.33	0.46	-2.32	-2.55	-9.58	-3.60			
16. Cattle Live	2.16	0.19	1.87	3.40	0.12	-2.41	1.58	-1.61	0.09	-2.28	-3.05	4.17	1.22	6.55	-6.37	3.79	5.06	9.46			
17. Hogs	2.53	0.31	0.76	1.29	4.93	-5.70	-3.35	-4.08	5.58	0.55	-3.58	2.57	-5.34	4.43	-0.43	-1.52	-4.09	-0.63			
18. Cattle Feeder	3.70	0.00	-2.62	9.86	0.18	-3.78	3.82	-2.49	-0.72	-2.28	-2.81	4.01	0.79	4.63	-4.29	0.77	-0.89	-2.00			
<i>Category IV: Energy</i>																					
19. Gasoil	-5.49	0.94	-18.6	-11.8	4.19	6.61	1.87	-0.17	-1.36	4.37	-0.84	-2.10	9.83	10.1	-15.5	1.62	2.90	-16.2			
20. Oil (Crude)	-7.14	0.95	-25.5	-21.9	3.04	10.2	0.34	-0.61	-0.23	1.91	0.12	-1.30	9.94	8.99	-14.5	-2.22	-6.25	-4.28			
21. Gasoline	-4.78	0.97	-25.9	-14.5	2.62	8.79	3.76	4.13	-5.85	2.84	-2.48	0.33	8.20	11.6	-16.3	-1.31	-9.90	-8.81			
22. Gas	-6.47	0.94	-29.7	-24.8	8.94	9.98	-2.88	5.17	-1.81	4.08	-6.03	2.43	6.86	10.4	-14.2	-1.64	-4.69	-1.88			
<i>Category V: Metals</i>																					
23. Silver	0.16	0.99	8.95	16.6	2.79	-15.5	10.6	-8.87	-0.47	4.39	-0.95	-2.02	-7.65	-1.67	6.09	2.90	16.7	14.0			
24. Gold	-2.52	0.99	12.8	19.8	0.82	-15.0	9.56	-1.19	-6.16	2.64	-1.12	-0.74	-8.92	-13.1	16.5	2.92	7.56	2.93			
25. Palladium	2.06	0.99	10.0	20.7	6.61	-16.3	11.6	-6.83	-3.06	-4.55	0.14	2.71	-9.47	-3.75	9.19	6.16	2.87	16.2			
26. Platinum	0.90	0.99	8.13	9.10	5.99	-14.3	7.05	-5.64	-0.42	0.57	-0.53	0.10	-6.79	-1.86	5.72	6.81	12.6	21.5			
27. Copper	10.4	0.99	3.16	10.6	4.38	-9.59	6.59	-2.52	-1.60	-0.04	-6.27	5.54	2.28	0.27	-1.30	6.64	13.0	8.88			
Average	1.47	0.68	-1.15	3.01	2.28	-3.42	3.07	-1.45	-0.92	0.34	-1.20	0.89	1.12	1.32	-1.91	0.84	-0.62	-1.73			

Note. This table reports the linear correlation coefficient between daily commodity futures returns and DJIA returns using all available data. The first column (ρ) reports the full sample correlation while all other columns report the difference in correlation between the subsample under consideration (e.g. start of recession) and its complement (e.g. all periods except start of recession). Column DCC reports the estimated autoregressive parameter of the correlation equation of the GARCH(1,1)-DCC(1,1) model. Business cycle and monetary regimes are as described in the text. Classifications on the level of volatility are based on univariate GARCH estimates, with low, medium, and high volatility regimes defined as periods where the corresponding volatility estimates are in the lower third, middle third, and upper third quantile of the distribution respectively. Boldface figures indicate 5% significance levels based on the bootstrap.

Table 5: Kendall's rank correlation with DJIA

	τ	Business cycle				Monetary regime				Equity vol			Commodity vol			Frequency		
		SR	ER	SE	EE	EM	RM	AM	Low	Med	High	Low	Med	High	1W	1M	3M	
<i>Category I: Grains and oil seeds</i>																		
1. Corn	0.50	-2.80	11.1	0.10	-1.91	6.06	-2.08	-2.92	0.84	-0.66	0.02	4.39	2.82	-6.71	3.97	-1.69	1.63	
2. Oats	1.00	0.05	4.80	2.32	-2.97	-0.55	0.55	-0.71	1.06	-0.65	-0.07	1.09	-1.71	0.44	0.98	0.19	1.13	
3. Soybeans	2.36	1.35	3.35	4.43	-4.96	3.63	-3.36	-0.91	-2.64	1.82	1.63	0.87	2.12	-2.35	4.49	-1.64	9.70	
4. Wheat	1.47	0.99	4.53	0.28	-2.64	3.60	-4.95	1.12	-0.61	0.45	0.49	4.37	0.45	-4.28	2.32	3.99	0.66	
5. Flaxseed	0.89	-2.33	2.95	4.16	-4.67	2.57	-0.56	-0.73	2.88	-3.92	-0.12	2.04	0.11	-2.41	1.69	-3.66	-9.16	
6. Rapeseed	2.03	0.17	9.37	6.39	-9.46	7.84	-6.70	-1.28	-0.18	-0.22	0.07	0.50	0.90	-1.45	4.00	-0.78	4.74	
<i>Category II: Softs</i>																		
7. Azuki Beans	-2.97	15.9	10.8	-1.84	-2.99	4.04	-9.88	1.21	-3.37	5.64	-1.69	-0.01	-3.68	2.61	0.19	-6.51	0.40	
8. Rough Rice	2.23	-3.02	7.20	6.35	-6.46	0.85	-6.10	4.38	5.13	-3.79	-0.13	2.02	-5.98	3.38	-1.03	2.35	21.1	
9. Cocoa	1.77	6.03	6.67	-2.75	-1.15	4.82	-3.82	-0.54	0.00	1.95	-2.23	1.10	-3.49	2.32	-0.63	-8.47	-23.2	
10. Sugar	0.59	5.80	1.49	-1.34	-0.37	1.26	0.67	-2.11	0.04	-2.78	2.93	0.25	0.94	-1.00	-1.51	3.71	-3.45	
11. Orange Juice	2.09	3.46	7.90	0.66	-4.82	3.88	-0.14	-2.78	-4.04	1.44	2.29	0.22	-1.03	1.41	-3.91	-1.61	-16.4	
12. Coffee	0.32	2.95	-2.35	0.94	-1.67	2.73	3.52	-3.65	-1.46	-3.52	3.91	-0.56	3.34	-2.74	0.48	-6.85	-6.57	
13. Cotton	3.16	0.69	8.01	-0.08	-2.58	1.72	1.22	-2.47	-1.49	-2.70	4.30	3.48	-2.77	-0.80	0.82	-3.40	-0.31	
14. Lumber	5.94	3.83	-1.95	5.79	-7.39	5.58	-4.72	0.38	1.19	-1.69	1.64	0.23	0.16	-0.85	2.96	3.80	14.6	
<i>Category III: Meat and livestock</i>																		
15. Pork Bellies	0.64	-1.33	6.04	4.41	-6.99	2.83	-7.61	4.33	2.07	-2.29	0.21	0.23	-2.22	1.93	-1.72	-13.4	7.16	
16. Cattle Live	1.28	4.08	6.84	1.74	-5.18	4.55	-6.36	1.02	-2.56	0.55	2.01	-0.54	3.65	-3.16	2.74	2.04	5.11	
17. Hogs	0.71	-3.13	7.51	4.34	-5.20	-0.09	-4.73	4.50	-0.59	-2.84	2.87	-2.87	1.60	0.89	0.70	-6.40	-5.68	
18. Cattle Feeder	1.58	3.24	3.92	1.43	-2.42	3.81	-3.49	0.19	-1.28	2.26	-0.09	0.31	-1.02	1.20	0.69	3.18	8.20	
<i>Category IV: Energy</i>																		
19. Gasoil	-1.08	-4.99	2.97	1.85	-2.30	5.91	-3.89	-1.61	-0.04	0.36	-1.48	5.72	2.34	-8.21	3.98	3.06	-6.76	
20. Oil (Crude)	-3.25	-9.90	6.16	-0.94	1.46	3.65	-0.37	-2.03	-1.62	3.17	-2.40	1.47	3.24	-4.46	0.79	-4.03	6.70	
21. Gasoline	-0.22	-5.12	9.16	-1.43	0.16	5.99	4.22	-7.19	-0.73	0.72	-0.27	0.91	5.02	-5.18	1.43	1.06	10.2	
22. Gas	-2.02	-10.3	1.68	5.28	-3.62	4.80	-4.31	-1.69	-0.07	4.34	-0.73	-0.55	8.07	-7.75	5.92	-4.71	-3.69	
<i>Category V: Metals</i>																		
23. Silver	-1.16	8.03	13.2	-1.54	-6.16	4.52	-4.36	0.97	1.49	-0.95	-1.12	-5.61	2.52	3.01	2.41	6.65	2.59	
24. Gold	-5.17	14.9	7.62	1.55	-6.74	2.56	-2.31	0.72	2.99	4.08	-4.48	-4.89	-0.94	7.09	3.38	0.72	3.50	
25. Palladium	0.38	11.9	9.92	2.22	-7.77	7.95	-1.07	-3.23	-3.44	0.46	4.12	-2.76	-7.73	9.73	2.88	3.19	-0.38	
26. Platinum	0.07	11.8	8.91	3.12	-9.56	2.72	-4.35	2.70	2.25	-1.23	-0.60	-3.62	-2.09	4.99	7.38	11.1	9.93	
27. Copper	5.02	2.46	6.24	2.02	-6.64	2.65	1.70	-2.42	0.54	-4.31	4.37	2.74	-1.16	-1.71	6.47	13.8	22.1	
Average	0.67	2.03	6.07	1.83	-4.26	3.70	-2.71	-0.55	-0.13	-0.16	0.57	0.39	0.13	-0.52	1.92	-0.16	2.00	

Note. This table reports Kendall's Tau between daily commodity futures returns and DJIA returns using all available data. The first column (" τ ") reports the full sample Kendall's Tau while all other columns report the difference in Kendall's Tau between the subsample under consideration (e.g. start of recession) and its complement (e.g. all periods except start of recession). Business cycle and monetary regimes are as described in the text. Classifications on the level of volatility are based on univariate GARCH estimates, with low, medium, and high volatility regimes defined as periods where the corresponding volatility estimates are in the lower third, middle third, and upper third quantile of the distribution respectively. Boldface figures indicate 5% significance levels based on the bootstrap.

Table 6: Correlation with 10-year US bonds

	ρ	Business cycle					Monetary regime					Bond vol					Commodity vol					Frequency		
		DCC	SR	ER	SE	EE	EM	RM	AM	Low	Med	High	Low	Med	High	Low	Med	High	IW	1M	3M			
<i>Category I: Grains and oil seeds</i>																								
1. Corn	-0.42	0.54	2.62	3.78	1.54	-4.15	0.21	2.85	-2.48	-3.22	-3.87	5.37	-1.82	0.51	0.61	-2.67	-7.85	-11.4						
2. Oats	1.20	0.99	5.84	3.44	2.06	-6.42	-1.49	2.47	-0.87	-3.22	-1.84	3.68	0.09	-0.17	0.16	0.63	-2.26	-6.33						
3. Soybeans	0.79	0.98	6.15	3.52	1.77	-5.48	1.39	2.51	-3.01	-6.08	-6.60	9.71	-4.67	0.05	2.84	-0.40	-8.13	-12.5						
4. Wheat	0.76	0.98	-0.33	0.81	2.11	-2.42	-2.43	1.91	0.32	-3.60	-2.57	4.63	0.85	-1.67	0.96	-2.79	-0.92	-2.40						
5. Flaxseed	0.66	0.75	2.49	5.54	3.04	-6.43	2.61	-3.00	0.66	-6.22	-2.80	7.01	0.32	2.67	-2.39	0.28	-3.65	-15.1						
6. Rapeseed	1.38	0.99	2.36	6.06	5.00	-8.78	3.27	0.34	-2.64	-7.00	-4.70	9.18	3.86	-0.95	-1.78	-1.93	-6.59	-16.9						
<i>Category II: Softs</i>																								
7. Azuki Beans	-0.92	0.99	5.52	31.8	7.88	-14.0	4.74	-0.71	-3.26	-2.90	1.51	1.07	1.57	3.91	-4.78	4.02	2.65	-4.47						
8. Rough Rice	1.12	0.30	10.8	-6.47	1.56	-2.67	-5.20	-4.32	6.45	-4.97	0.34	3.35	-5.12	-4.50	6.84	1.55	2.40	-7.04						
9. Cocoa	0.56	0.75	1.46	2.51	-1.47	-0.13	0.60	0.14	-0.62	-3.88	0.28	2.37	1.55	-0.58	-0.60	-1.25	-6.29	-15.1						
10. Sugar	0.56	0.96	6.05	2.31	-0.13	-3.67	-1.70	6.16	-3.28	-3.10	-1.22	3.00	-6.27	1.05	3.21	-0.32	-4.18	-1.14						
11. Orange Juice	-0.19	0.77	-1.23	0.99	5.96	-6.04	-4.12	2.10	1.51	-1.76	1.98	-0.60	1.18	-1.46	0.46	-0.52	-8.34	-17.4						
12. Coffee	-2.80	0.00	2.54	9.49	0.43	-4.52	6.65	-2.24	-2.92	-0.69	-0.95	1.07	4.71	-1.52	-2.33	-0.11	-3.37	-10.6						
13. Cotton	-1.01	0.95	6.39	3.03	-2.00	-2.03	-1.78	4.14	-1.95	-2.29	-4.04	4.74	3.07	2.11	-4.00	-4.14	-10.9	-14.0						
14. Lumber	8.52	0.99	4.56	0.99	1.00	-3.60	0.34	1.22	-1.26	-6.00	-6.28	9.51	-0.55	1.14	-0.64	-1.14	-7.99	-7.08						
<i>Category III: Meat and livestock</i>																								
15. Pork Bellies	0.66	0.63	7.59	-2.07	4.04	-6.78	2.40	-3.86	1.15	-1.87	-1.37	2.34	2.03	-1.62	0.05	-1.48	-4.65	-11.2						
16. Cattle Live	0.03	0.98	13.6	3.51	2.21	-10.4	5.93	-4.82	-0.92	-4.82	2.34	1.50	0.17	0.25	-0.28	-0.08	-10.5	-20.6						
17. Hogs	0.91	0.99	11.3	0.40	3.47	-9.30	4.57	-1.03	-2.86	-2.79	0.22	1.71	3.35	4.37	-5.73	0.38	-2.61	-13.0						
18. Cattle Feeder	2.23	0.99	13.5	0.13	2.07	-9.35	3.58	-0.84	-2.19	-3.78	-1.42	4.05	-2.07	0.56	0.81	-0.13	-12.2	-8.17						
<i>Category IV: Energy</i>																								
19. Gasoil	-4.60	0.81	-3.76	-2.74	1.80	1.23	-3.71	5.93	-0.95	-1.17	-4.57	4.63	4.72	5.29	-8.74	1.90	-0.69	-6.19						
20. Oil (Crude)	-7.58	0.75	-23.5	-7.87	2.46	4.09	-2.96	4.88	-0.16	-3.91	-0.63	3.33	7.57	4.29	-9.02	0.80	-5.40	-20.3						
21. Gasoline	-5.10	0.88	-19.5	-14.6	4.13	3.74	-4.01	6.32	-0.39	0.78	-3.56	2.43	4.78	5.77	-8.34	2.76	-5.16	-12.5						
22. Gas	-6.49	0.95	-19.6	-17.8	7.44	3.27	-1.67	-4.25	3.58	-1.53	-6.81	7.76	6.53	5.26	-9.23	-2.26	7.17	0.15						
<i>Category V: Metals</i>																								
23. Silver	-2.54	0.99	7.01	18.1	6.46	-20.4	10.8	-6.49	-3.36	-5.12	-4.80	6.73	-1.71	-5.16	5.05	-1.78	-11.2	-34.0						
24. Gold	0.62	0.99	4.32	14.8	10.3	-22.3	10.7	-2.25	-7.00	-12.0	-5.49	11.5	-11.5	-7.52	12.2	-2.60	-3.12	-6.57						
25. Palladium	-0.53	0.99	6.86	3.41	-13.0	-13.0	7.04	-4.93	-1.99	-3.47	-0.64	2.79	-9.76	-1.49	6.72	-0.94	-6.86	-1.39						
26. Platinum	-1.44	0.99	5.49	14.9	4.29	-14.7	9.88	-4.49	-4.16	-6.52	-4.43	7.33	-7.80	-5.09	8.82	-0.39	-4.52	-8.72						
27. Copper	-1.04	0.99	4.72	17.5	5.84	-14.9	9.14	-3.95	-2.84	-2.74	-2.25	3.55	-4.39	1.18	1.93	-1.73	-0.60	-8.40						
Average	-0.54	0.85	2.34	3.99	3.21	-6.63	2.03	-0.23	-1.31	-3.84	-2.38	4.58	-0.34	0.25	-0.26	-0.53	-4.66	-10.8						

Note. See table 4.

Table 7: Kendall's rank correlation with 10-year US bonds

	Business cycle				Monetary regime				Bond vol				Commodity vol				Frequency				
	τ	SR	ER	SE	EE	EM	RM	AM	Low	Med	High	Low	Med	High	Low	Med	High	1W	1M	3M	
<i>Category I: Grains and oil seeds</i>																					
1.	Corn	-1.95	3.16	8.74	2.86	-5.23	6.04	-0.17	0.23	2.78	-4.13	4.78	-2.85	-0.40	3.35	-2.34	-1.26	-4.46			
2.	Oats	-1.58	6.55	3.26	3.92	-6.66	3.04	2.24	-2.33	-4.66	-0.39	4.95	-2.38	-0.70	3.04	1.48	-3.34	-0.98			
3.	Soybeans	-2.12	1.67	7.58	1.90	-4.75	2.69	0.17	-1.57	-1.15	-1.78	2.34	-3.09	2.13	1.61	0.07	1.04	-19.7			
4.	Wheat	0.66	6.60	-3.11	4.56	-4.97	-0.76	0.96	1.76	-3.36	-2.09	5.82	0.55	-1.99	1.42	-3.63	-4.30	-3.23			
5.	Flaxseed	-0.56	7.92	6.92	5.65	-8.82	4.23	-3.53	1.79	-3.16	-1.16	5.55	2.01	1.57	-3.10	-0.71	1.82	-16.1			
6.	Rapeseed	-0.76	4.38	6.64	5.06	-8.95	3.73	0.83	-1.92	-5.74	-1.11	7.44	1.93	-1.40	1.86	-0.98	-2.52	-10.5			
<i>Category II: Softs</i>																					
7.	Azuki Beans	0.17	5.73	12.2	4.57	-6.26	3.60	3.14	-4.33	-3.53	-0.50	3.65	-0.34	4.95	-5.05	5.44	-5.38	-4.06			
8.	Rough Rice	-0.75	8.10	-3.56	-0.16	-0.89	-3.22	-3.20	4.70	-4.23	-0.18	4.40	-0.08	-5.98	5.91	1.97	5.38	11.9			
9.	Cocoa	-0.66	-3.04	-1.93	2.89	-1.18	0.41	3.09	-1.57	-1.83	-0.09	1.77	0.65	1.50	-2.61	-0.63	3.69	-1.94			
10.	Sugar	-0.90	-0.52	8.19	-2.82	0.74	-2.47	4.45	-1.15	-0.25	-0.28	0.34	-0.85	-1.16	2.29	-0.41	0.90	-3.05			
11.	Oranges Juice	0.06	6.83	-1.47	3.59	-3.61	-2.42	-0.25	1.31	-3.30	3.70	-0.23	3.79	-1.74	-1.55	1.79	-4.95	-6.73			
12.	Coffee	-1.34	-2.97	0.11	3.73	-3.74	0.96	0.26	-0.37	-0.11	-0.02	-0.13	3.59	1.00	-4.15	-2.98	-5.50	-1.02			
13.	Cotton	0.45	4.48	-3.32	0.41	0.53	-4.11	2.14	1.34	-0.43	-0.03	1.45	1.78	-0.05	-3.94	-5.25	-3.80	-3.80			
14.	Lumber	4.80	8.17	1.01	1.76	-4.23	2.53	1.61	-2.28	-5.22	-2.29	7.39	-0.62	1.06	-0.47	-3.37	-9.13	-9.84			
<i>Category III: Meat and livestock</i>																					
15.	Pork Bellies	-0.69	5.89	1.33	3.32	-4.33	3.80	-5.53	2.95	-0.22	0.91	0.26	0.48	1.26	-1.56	0.23	-2.52	-9.56			
16.	Cattle Live	-0.02	7.12	2.83	3.07	-4.27	4.52	-0.48	-1.47	-0.96	-0.31	2.20	2.23	-0.21	-0.46	-2.29	-6.44	-12.2			
17.	Hogs	0.25	4.11	-2.48	4.68	-4.61	3.11	-3.99	2.08	2.08	-1.23	-0.72	6.04	0.46	-6.55	-2.06	-3.50	-11.9			
18.	Cattle Feeder	1.04	12.9	-6.01	3.19	-3.79	4.55	-2.86	-1.04	-2.41	0.15	1.77	0.55	0.14	-0.64	-0.50	-6.12	-7.10			
<i>Category IV: Energy</i>																					
19.	Gasoil	-1.06	-1.05	-4.38	-1.50	2.58	-0.94	2.35	0.24	-1.83	0.87	0.34	-0.20	4.64	-3.19	-3.13	3.39	-17.8			
20.	Oil (Crude)	-1.13	-16.4	-8.40	-2.27	4.59	-1.38	-2.28	2.42	-0.60	-0.80	0.89	-4.34	2.66	0.72	-1.74	-0.42	3.45			
21.	Gasoline	-2.01	-13.5	-12.4	1.89	1.04	-1.35	3.09	-0.76	-2.05	0.40	1.97	3.03	2.60	-5.31	4.54	-0.97	-6.85			
22.	Gas	-3.83	-14.0	-13.3	1.47	3.10	3.64	-5.79	1.98	-0.13	1.46	0.22	3.36	1.24	-4.05	-4.63	1.88	14.0			
<i>Category V: Metals</i>																					
23.	Silver	-3.25	7.03	6.03	4.53	-8.05	5.59	-0.93	-2.47	-4.63	-1.56	4.92	-1.84	-0.49	2.54	-2.83	-6.70	-17.0			
24.	Gold	-3.01	7.90	7.14	8.86	-11.6	9.50	-0.89	-5.88	-4.84	-0.04	6.30	-7.39	-0.65	7.49	1.00	1.89	3.85			
25.	Palladium	-3.72	9.98	8.72	2.89	-5.37	3.02	0.79	-1.18	0.70	-2.61	2.39	-3.46	0.94	2.34	5.59	-1.70	-9.79			
26.	Platinum	-3.40	8.58	6.19	4.95	-8.30	4.95	0.23	-2.44	0.96	-4.04	2.77	-5.42	2.59	1.04	1.45	-3.50	-15.3			
27.	Copper	-2.21	8.19	10.1	4.71	-8.16	1.04	1.49	0.59	-1.14	-4.59	6.08	-2.20	3.13	-0.11	-5.37	-6.02	-17.3			
	Average	-1.02	3.11	1.36	2.88	-3.90	2.01	-0.11	-0.35	-1.83	-0.81	2.92	-0.19	0.63	-0.34	-0.71	-2.15	-6.56			

Note. See table 5.

Table 8: Symmetrized Joe-Clayton upper and lower tail dependence parameters

		Grains and oil seeds						Softs						Meat and livestock						Energy						Metals											
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
1	-	45	48	40	27	36	0	4	0	1	0	0	3	0	0	0	2	0	0	0	0	0	0	2	0	0	1	0									
2	45	-	34	33	20	26	0	3	0	0	0	2	0	0	0	0	1	0	0	0	0	0	0	3	0	0	1	0									
3	47	36	-	28	33	50	0	6	2	0	0	4	0	0	0	1	0	2	0	0	0	0	0	3	0	0	1	0									
4	39	32	29	-	16	20	0	4	0	1	0	2	0	0	0	1	1	2	0	0	0	0	0	2	0	0	0	0									
5	21	18	31	14	-	47	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0									
6	33	25	48	21	46	-	0	3	0	0	0	3	0	0	0	0	0	1	0	0	0	0	0	3	0	0	0	0									
7	0	0	0	0	0	0	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0									
8	4	2	4	2	0	3	0	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0									
9	0	0	0	0	0	0	0	0	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	1	0	1	1									
10	0	0	1	0	1	1	0	0	0	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0									
11	0	0	0	0	0	0	0	0	0	0	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0									
12	0	0	0	0	0	0	0	0	1	0	0	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0									
13	5	3	6	3	0	2	0	0	0	0	0	0	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0									
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-	0	0	0	0	0	0	0	0	0	0	0	0	0									
15	2	2	2	1	1	1	0	0	0	0	0	0	1	0	0	-	11	45	9	0	0	0	0	0	0	0	0	0									
16	1	2	4	1	3	2	0	0	0	0	0	0	0	0	0	16	-	22	57	0	0	0	0	0	0	0	0	0									
17	1	2	2	1	1	1	0	0	0	0	0	0	1	0	0	45	28	-	19	0	0	0	0	0	0	0	0	0									
18	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	15	64	27	-	0	0	0	0	0	0	0	0	0									
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-	67	58	32	0	0	0	0	0									
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	69	-	60	27	0	1	0	0	0									
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	62	66	-	24	0	1	0	0	0									
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	34	35	27	-	0	1	0	0	0									
23	3	5	9	4	6	5	0	0	1	3	0	0	1	0	1	1	1	1	1	1	2	1	1	-	50	24	37	13									
24	1	3	8	3	4	5	0	0	2	4	0	0	0	0	0	0	0	0	0	2	3	2	3	54	-	25	45	7									
25	1	1	3	1	2	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	33	32	-	37	3									
26	1	3	5	2	4	3	0	0	1	3	0	0	0	0	0	0	0	0	0	0	0	0	0	43	50	45	-	9									
27	1	2	4	2	4	3	0	0	0	1	0	0	1	1	1	0	0	0	1	0	0	0	0	16	13	10	12	-									

Note. This table reports MLE estimates of the SJC lower (upper) tail dependence parameter τ_L (τ_U) in the lower (upper) diagonal, using all available data.

Table 9: Average of 10 most extreme commodity, bond, and equity returns (1987–2005)

	commodity returns		bond returns when commodity returns are		equity returns when commodity returns are		bond returns are		equity returns are	
	low	high	low	high	low	high	low	high	low	high
<i>Category I: Grains and oil seeds</i>										
1. Corn	-5.45	5.55	-0.10	-0.23	0.68	0.03	0.37	0.25	-0.79	0.70
2. Oats	-7.94	7.70	-0.10	0.20	0.33	-0.06	0.06	0.06	-0.39	0.63
3. Soybeans	-5.95	5.13	0.19	-0.07	0.30	0.53	0.03	0.66	-0.56	0.21
4. Wheat	-5.60	6.66	-0.09	-0.10	-0.27	1.18	1.07	0.20	-0.96	0.93
5. Flaxseed	-4.21	4.83	0.34	-0.02	0.12	0.30	-0.10	0.20	-0.77	-0.00
6. Rapeseed	-4.63	4.16	0.08	-0.07	-0.04	0.09	-0.32	-0.28	-0.91	0.10
<i>Category II: Softs</i>										
7. Azuki Beans	-5.02	4.62	0.34	0.07	0.14	0.13	-0.21	-0.47	0.33	-0.25
8. Rough Rice	-6.66	6.78	0.09	0.15	0.54	-0.43	0.45	1.01	0.21	-0.49
9. Cocoa	-7.95	9.02	-0.12	-0.10	0.35	0.27	-0.19	0.59	0.87	-0.05
10. Sugar	-10.22	8.66	-0.10	-0.27	0.14	-0.24	-0.47	-0.86	0.41	-1.47
11. Orange Juice	-9.32	13.13	0.21	-0.21	0.15	0.74	-0.49	-0.26	-0.44	-0.43
12. Coffee	-12.05	15.10	-0.04	-0.29	0.52	0.26	1.42	-0.93	-0.69	1.25
13. Cotton	-6.44	6.75	0.16	-0.39	-0.33	-0.12	-0.20	-0.45	-0.93	0.47
14. Lumber	-4.36	4.34	-0.04	-0.18	-0.21	0.25	1.02	-0.06	-1.80	-0.20
<i>Category III: Meat and livestock</i>										
15. Pork Bellies	-6.54	6.71	0.12	-0.24	0.18	0.06	0.55	-0.36	-0.37	-1.29
16. Cattle Live	-2.70	2.96	-0.06	-0.11	0.47	-0.57	0.33	-0.35	-0.55	-0.36
17. Hogs	-6.04	5.67	-0.02	-0.05	-0.76	0.27	0.02	0.04	-1.22	-0.12
18. Cattle Feeder	-2.74	2.66	-0.05	-0.36	0.05	-0.44	0.34	-0.15	-0.68	-0.30
<i>Category IV: Energy</i>										
19. Gasoil	-13.27	9.28	0.24	-0.07	1.40	-0.76	-1.37	0.30	-0.53	-0.00
20. Oil (Crude)	-14.99	10.57	-0.05	-0.28	1.39	-0.70	-0.45	0.10	-0.19	-0.06
21. Gasoline	-12.42	9.17	-0.16	-0.13	1.24	-0.80	-0.93	0.05	-0.68	0.46
22. Gas	-16.31	13.72	0.35	-0.15	0.01	-0.85	-0.67	0.14	0.43	0.05
<i>Category V: Metals</i>										
23. Silver	-8.90	6.83	0.81	-0.12	1.29	-0.39	0.19	-0.85	0.79	-2.18
24. Gold	-5.19	4.64	1.00	-0.17	1.36	0.23	-0.40	0.16	0.87	-0.88
25. Palladium	-10.56	11.45	-0.02	0.13	-0.38	0.70	0.38	-0.65	1.12	-0.88
26. Platinum	-6.10	6.22	0.19	-0.17	0.35	-0.94	-0.43	-0.77	1.19	-0.76
27. Copper	-9.01	6.90	0.78	-0.03	0.33	0.52	0.44	-1.19	-0.98	-0.41
Average	-7.80	7.38	0.15	-0.12	0.35	-0.03	0.02	-0.14	-0.27	-0.20

Note. This table reports the average of the 10 most extreme returns in percentages over the period 1987 – 2005.

Table 10: SJC copula tail dependence parameters

	Equity tail dependence												Bond tail dependence											
	daily		weekly		monthly		quarterly		daily		weekly		monthly		quarterly									
	τ_U	τ_L	τ_U	τ_L	τ_U	τ_L	τ_U	τ_L	τ_U	τ_L	τ_U	τ_L	τ_U	τ_L	τ_U	τ_L								
<i>Category I: Grains and oil seeds</i>																								
1. Corn	0.0	0.0	0.0	0.0	0.0	1.3	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0								
2. Oats	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0								
3. Soybeans	0.0	0.0	0.0	0.2	0.0	2.3	0.0	3.2	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0								
4. Wheat	0.0	0.0	0.0	0.0	0.0	1.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0								
5. Flaxseed	0.0	0.0	0.0	0.0	0.0	0.3	0.2	1.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0								
6. Rapeseed	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0								
<i>Category II: Softs</i>																								
7. Azuki Beans	0.0	0.0	0.0	0.0	0.0	7.2	0.0	10.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	14.5								
8. Rough Rice	0.0	0.0	0.0	0.7	0.0	2.0	0.0	27.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	3.8								
9. Cocoa	0.0	0.0	0.0	0.0	1.7	0.2	3.5	12.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	6.2								
10. Sugar	0.0	0.0	0.0	0.0	0.0	0.0	0.0	7.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0								
11. Orange Juice	0.0	0.0	0.0	0.0	0.0	2.1	0.1	0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.4	0.1								
12. Coffee	0.0	0.0	0.0	0.0	0.0	0.0	0.0	2.2	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0								
13. Cotton	0.0	0.0	0.0	0.0	0.0	4.2	15.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	4.5	8.3								
14. Lumber	0.6	1.1	1.5	1.3	4.2	10.1	1.7	21.0	0.0	0.6	0.0	1.8	0.0	0.0	0.0	0.0								
<i>Category III: Meat and livestock</i>																								
15. Pork Bellies	0.0	0.0	0.0	0.0	0.0	3.1	0.0	0.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	3.7								
16. Cattle Live	0.0	0.0	0.0	0.0	0.0	1.5	1.2	0.4	0.0	0.0	0.0	0.0	0.0	0.0	1.0	8.3								
17. Hogs	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	17.0								
18. Cattle Feeder	0.0	0.0	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.8	0.0	0.0								
<i>Category IV: Energy</i>																								
19. Gasoil	0.0	0.0	0.0	0.0	0.0	0.0	25.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	7.3								
20. Oil (Crude)	0.3	0.0	2.3	0.0	4.1	3.1	0.0	3.0	0.2	0.0	1.0	0.0	0.0	5.4	0.4	11.3								
21. Gasoline	0.0	0.0	0.1	0.0	0.0	4.9	1.8	2.5	0.0	0.0	0.0	0.0	0.0	1.9	0.0	2.7								
22. Gas	0.0	0.1	1.3	0.0	5.6	0.0	0.0	10.2	0.0	0.0	0.5	0.0	0.0	0.0	0.0	5.6								
<i>Category V: Metals</i>																								
23. Silver	0.0	0.0	0.0	0.2	0.1	11.2	0.2	0.4	0.1	0.0	1.4	0.0	0.0	9.5	0.0	20.9								
24. Gold	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.6								
25. Palladium	0.0	0.0	0.0	0.5	0.0	0.0	0.0	9.8	0.0	0.0	0.0	0.0	0.0	4.9	0.0	0.1								
26. Platinum	0.0	0.0	1.4	0.0	0.0	11.5	0.0	28.2	0.0	0.0	0.8	0.0	0.0	3.7	0.0	0.8								
27. Copper	0.1	1.8	0.3	7.6	0.0	22.5	0.0	26.7	0.0	0.0	0.1	0.0	0.0	0.0	0.0	3.1								

Note. This table reports MLE estimates of the SJC lower (upper) tail dependence parameter τ_L (τ_U) using all available data at different sampling frequencies.

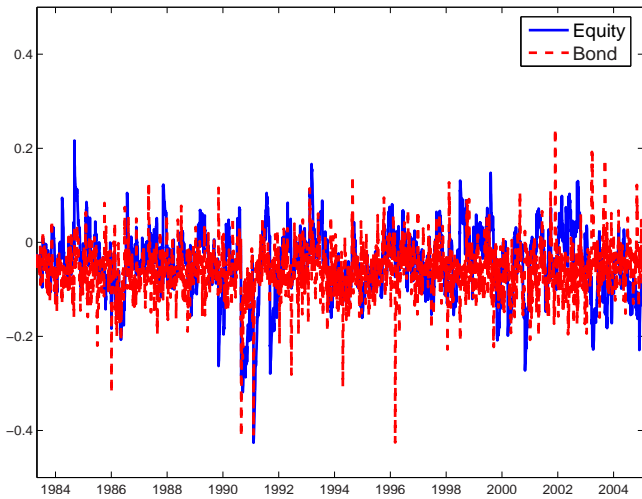
Table 11: Correlations with unexpected inflation rate (annual frequency)

	Unexpected inflation CPI						Unexpected inflation PPI (finished goods)						Unexpected inflation ECI					
	all			all less food & energy			all			all less food & energy			public			private		
	Ftr	Spt	Rll	Ftr	Spt	Rll	Ftr	Spt	Rll	Ftr	Spt	Rll	Ftr	Spt	Rll	Ftr	Spt	Rll
<i>Category I: Grains and oil seeds</i>																		
1. Corn	22.9	27.8	-11.9	22.9	38.4	-31.0	13.4	22.5	-18.2	12.6	26.8	-27.2	-30.6	-14.7	-21.4	32.6	37.7	-15.9
2. Oats	-23.7	-6.24	-27.1	-8.36	21.9	-40.9	-23.3	-20.2	-8.45	-9.99	17.5	-37.5	-1.33	-11.2	12.8	5.97	-8.90	22.5
3. Soybeans	23.2	14.3	21.2	39.3	29.7	23.1	31.0	22.8	19.8	39.0	29.9	22.1	-11.4	-22.9	27.8	41.6	31.1	15.7
4. Wheat	33.1	23.4	20.9	29.3	30.9	5.25	28.0	23.4	13.0	35.9	40.1	3.56	-39.8	-13.1	-39.7	13.8	27.7	-13.5
5. Flaxseed	43.4	-	-	42.3	-	-	37.2	-	-	42.3	-	-	43.8	-	-	14.2	-	-
6. Rapeseed	27.4	4.36	22.7	22.8	27.2	-8.24	30.1	8.26	20.9	33.2	28.1	1.26	12.7	2.98	26.2	-9.16	-8.15	-4.65
<i>Category II: Softs</i>																		
7. Azuki Beans	-11.9	-	-	-11.7	-	-	-1.04	-	-	-17.6	-	-	11.5	-	-	18.8	-	-
8. Rough Rice	12.4	-	-	21.2	-	-	3.18	-	-	-15.3	-	-	-40.5	-	-	27.4	-	-
9. Cocoa	-25.8	-13.2	-3.67	-11.3	8.90	-16.0	-40.2	-2.74	-23.2	-21.3	20.0	-33.3	-3.33	-12.3	18.8	-40.6	-32.1	-22.3
10. Sugar	45.4	55.7	-19.8	48.4	62.5	-26.8	30.5	39.1	-16.4	41.2	54.4	-24.9	12.2	5.61	10.9	34.7	17.8	28.8
11. Orange Juice	29.7	-	-	17.2	-	-	15.8	-	-	2.62	-	-	4.44	-	-	26.2	-	-
12. Coffee	1.44	-1.67	5.36	-11.6	-22.5	13.6	-2.04	-0.42	-3.19	-17.4	-20.8	-0.22	6.33	2.94	6.98	7.45	8.51	-2.53
13. Cotton	3.98	-3.39	15.0	-1.82	-5.38	7.76	12.8	6.20	12.2	-3.32	-8.71	11.8	8.93	9.44	-1.67	-6.70	-3.28	-6.09
14. Lumber	30.8	-6.46	55.0	27.6	-13.9	58.8	20.7	-3.70	36.2	-4.80	-28.2	26.0	4.63	-15.3	27.8	25.3	-4.59	49.8
<i>Category III: Meat and livestock</i>																		
15. Pork Bellies	0.84	2.08	-1.51	1.25	-10.6	15.3	2.24	12.8	-13.4	-26.5	-15.7	-16.1	13.9	32.3	-19.7	5.63	1.51	6.41
16. Cattle Live	32.4	44.4	5.26	7.95	-7.58	21.8	34.3	53.3	-1.33	-18.2	-10.4	-19.2	1.07	27.2	-18.6	14.7	21.1	3.43
17. Hogs	19.9	8.10	10.4	24.3	-12.4	40.9	9.42	19.3	-14.8	-13.0	-23.8	16.8	-13.2	31.3	-36.2	6.55	19.0	-7.81
18. Cattle Feeder	39.6	42.4	2.13	46.2	42.2	15.3	42.4	37.6	15.9	5.55	15.0	-15.5	4.32	19.9	-16.8	19.0	29.1	-4.51
<i>Category IV: Energy</i>																		
19. Gasoil	39.4	54.2	-4.84	4.77	4.42	2.11	44.6	61.1	-5.12	-7.96	-5.11	-6.34	24.4	38.1	-13.1	48.6	41.0	23.8
20. Oil (Crude)	48.4	64.0	2.64	-1.57	0.89	-4.44	49.6	66.1	2.04	-21.7	-23.5	-8.77	28.4	41.0	-3.90	48.4	41.5	34.3
21. Gasoline	33.7	57.4	-26.7	3.92	-3.04	11.6	37.1	61.7	-27.0	-6.40	-23.1	23.4	13.8	33.5	-25.3	52.1	50.0	18.7
22. Gas	58.2	64.3	-11.7	9.98	-9.97	46.5	55.4	70.5	-32.8	-15.6	-20.6	11.1	2.45	23.8	-43.1	46.2	47.2	5.86
<i>Category V: Metals</i>																		
23. Silver	43.8	42.8	26.0	37.3	38.6	10.1	34.2	29.8	40.6	19.5	16.5	25.9	-15.4	-19.7	10.3	47.7	49.5	21.8
24. Gold	50.4	47.3	17.7	46.7	48.0	-6.90	43.4	39.5	22.2	34.8	33.0	10.5	4.72	4.24	4.27	23.4	24.4	3.01
25. Palladium	0.50	-0.38	5.33	8.28	6.63	11.7	11.9	10.7	9.65	14.8	13.7	10.4	9.61	7.69	18.3	40.7	36.4	50.7
26. Platinum	46.7	22.3	44.7	40.0	23.0	33.6	46.0	34.2	29.7	44.0	27.4	34.6	13.6	7.08	17.2	44.3	49.8	-12.4
27. Copper	54.1	-	-	44.8	-	-	58.7	-	-	33.2	-	-	5.39	-	-	41.2	-	-
Average	25.2	24.7	6.69	18.5	13.5	8.32	23.2	26.9	2.65	5.91	6.48	0.38	2.62	8.09	-2.64	23.3	21.7	8.87
Equity	-33.0	-	-	-27.4	-	-	-32.2	-	-	-30.5	-	-	-	-31.0	-	-	10.3	-
Bonds	-52.9	-	-	-17.6	-	-	-36.5	-	-	-2.27	-	-	-	-24.8	-	-	-45.8	-

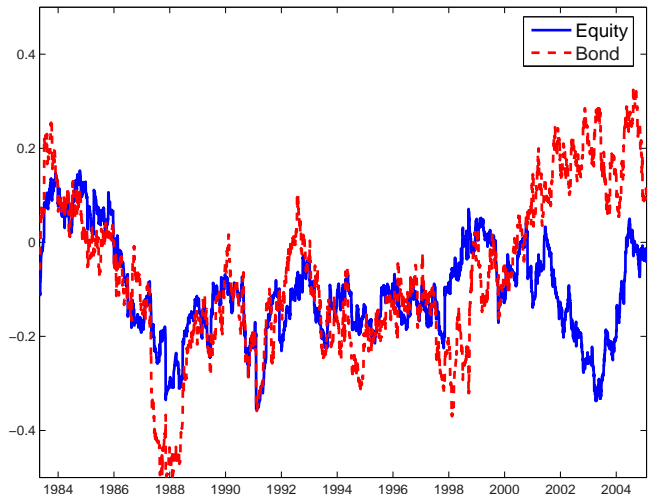
Note. This table reports Pearson's correlation between commodity futures returns and unexpected inflation at an annual frequency. Where spot prices are available, futures returns ("Ftr") are decomposed into a spot return component ("Spt") and a roll return component ("Rll") as detailed in Kat and Oomen (2006). The CPI and PPI data are from 1973 onwards while the ECI data are from 1982 onwards. Boldface figures indicate 5% significance based on the bootstrap.

Figure 1: Commodity correlations with Bond/Equity

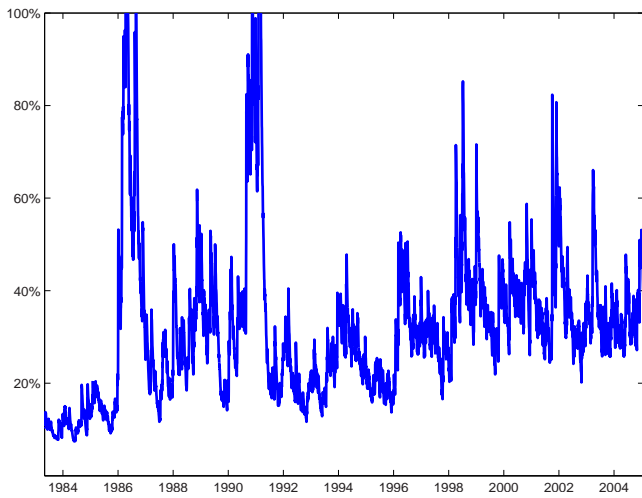
Panel A: oil correlations



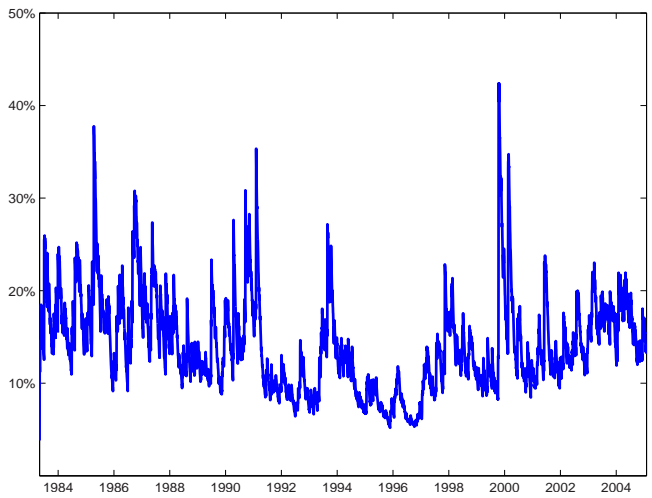
Panel B: gold correlations



Panel C: oil volatility



Panel D: gold volatility



Note. This figure plots daily bond (red dashed line) and equity (blue solid line) correlations with crude oil (Panel A) and gold (Panel B) obtained by fitting a GARCH(1,1)-DCC(1,1) model to data over the period 1983 – 2005. Panels C and D plot annualized crude oil and gold volatility.

Figure 2: Random drawings from various bivariate distributions with a correlation coefficient of 0.7

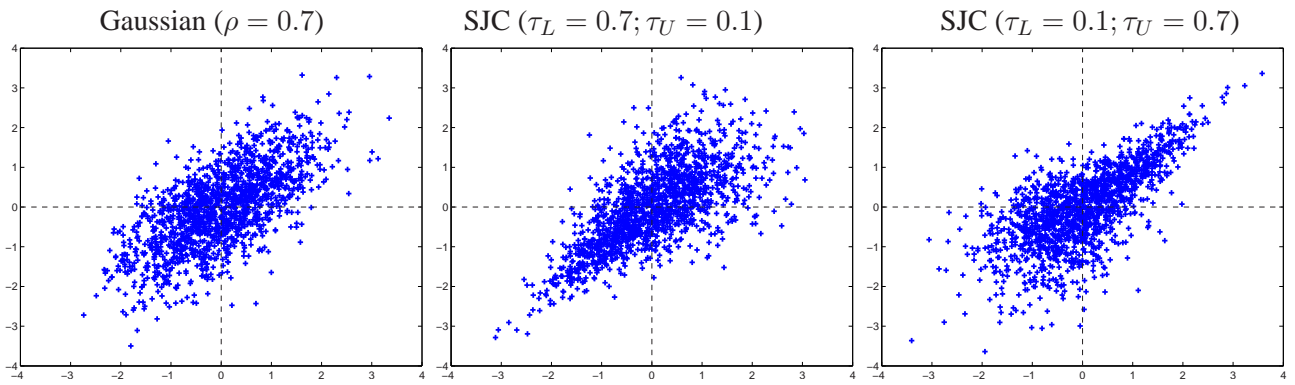
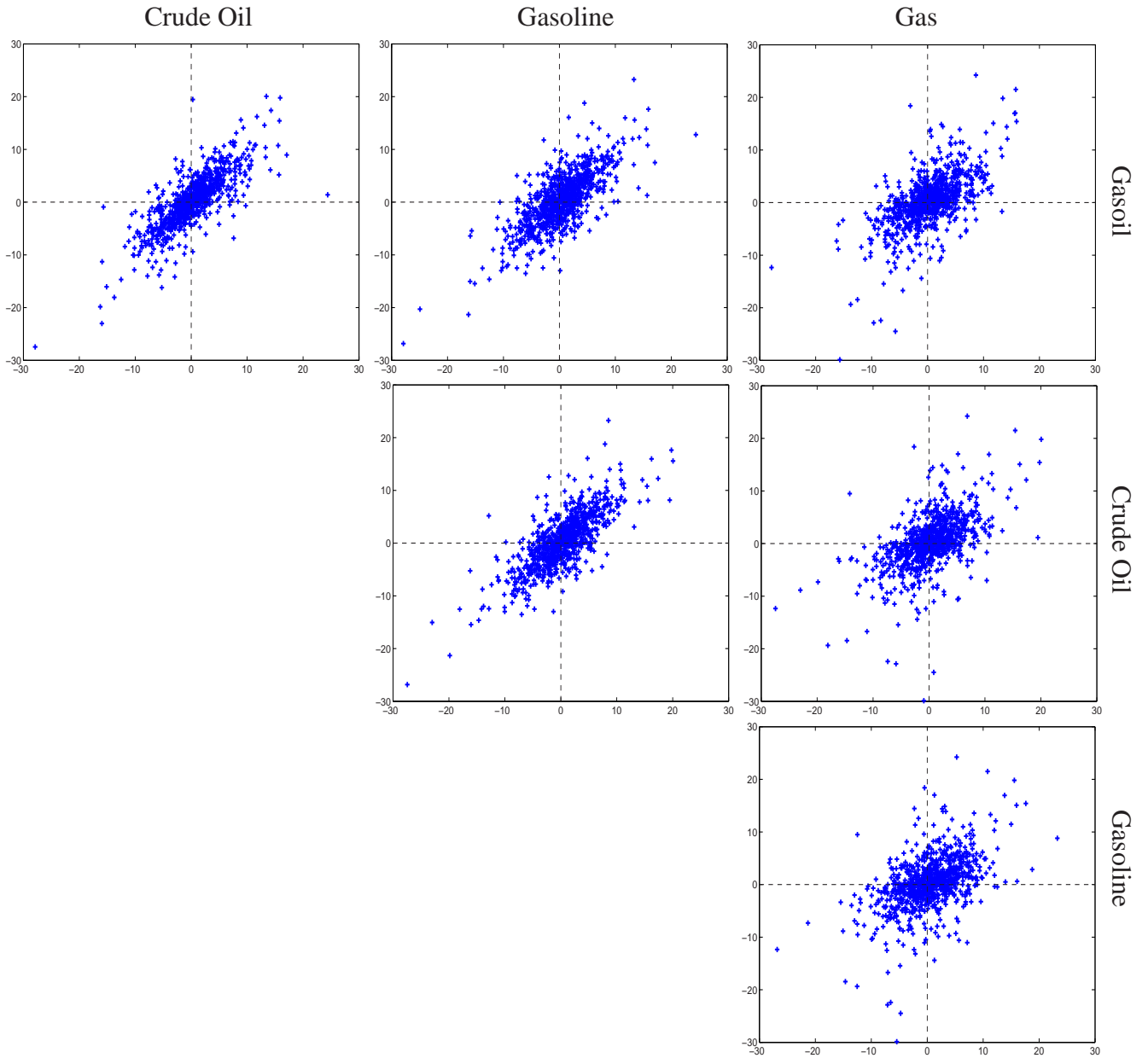
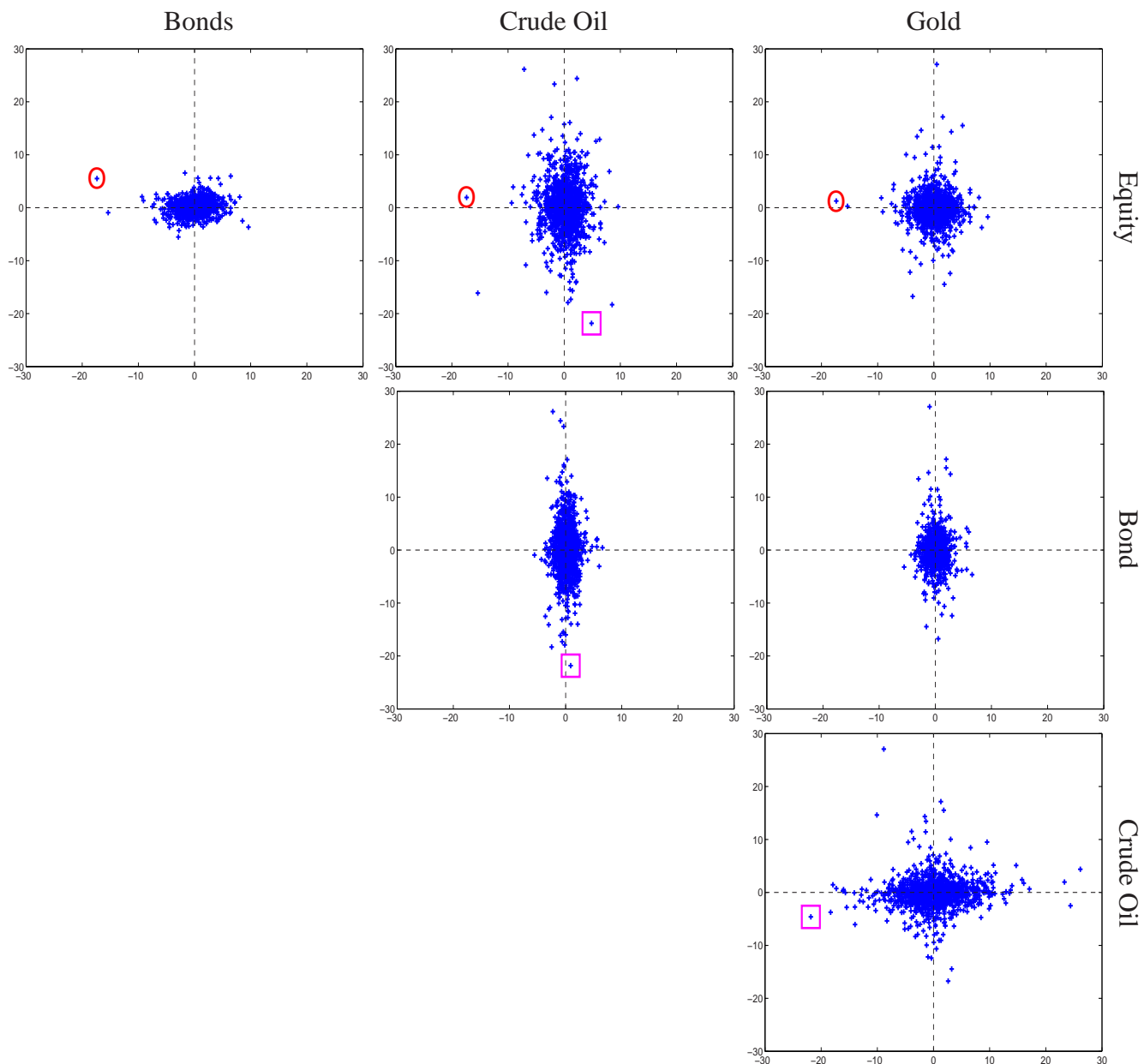


Figure 3: Scatterplot of weekly energy returns (1987 – 2005)



Note. Scatterplot of weekly returns in percentages for “energy” group components. The variables printed at the top (righthand-side) of the graph are on the vertical (horizontal) axis.

Figure 4: Scatterplot of weekly equity, bond, and commodity returns (1979 – 2005)



Note. Scatterplot of weekly returns in percentages for equities (DJIA), bonds (US gov 10Y), oil (crude), and gold. The variables printed at the top (righthand-side) of the graph are on the vertical (horizontal) axis. The red circle in the graphs involving equity indicates the October '87 stock market crash. The magenta square in the graphs involving crude oil indicates the January 1991 drop in oil prices following the US invasion of Iraq.